STEREOMETRY,

Or, the Art of

GAUGING

Made eaſie by the Help of a

SLIDING-RULE:

WHICH SHEWS

The Area's of Circles in Gallons, and the Square and Cube-Roots of any Number under 100000 by inſpection; and is alſo very uſeful in the ready Solution of many Queſtions in Arithmetick and Geometry. Particularly in Meaſuring of Timber, Board, Stone, Brick-work, Paving, Cieling, Flooring, Tyling, &c.

The Third Edition, carefully Corrected, and much Enlarged.

WITH AN

APPENDIX

OF

CONICK SECTIONS,

IN WHICH

The Nature and Original of ſeveral Solids frequently mentioned in Books of Gauging, is Explained, and their Magnitudes compared.

By *THO. EVERARD*, Gent.

London: Printed for Robert Clavel, at the Peacock in S. *Paul's* Church-Yard, and Christopher Wilkinſon at the *Flower de Luce* in *Little Britain*.

TO

The Right Honourable

CHARLES MOUNTAGUE, Esq.

One of the

Lords Commissioners

OF THE

TREASURY:

CHANCELLOR and Under-

TREASURER of the Exchequer:

ONE OF

His MAJESTYS

most Honorable Privy Council:

PRESIDENT

OF THE

ROYAL SOCIETY,

And one of the

MEMBERS of the

Honorable Hous of COMMONS,

THIS

TRACT

Is Humbly Dedicated by

Tho. Everard.

BRITISH MUSEUM 30 60 NOV

TO

The Right Honourable

CHARLES MOUNTAGUE, Esq;

One of the

Lords Commissioners

OF THE

TREASURY:

CHANCELLOR and Under-

TREASURER of the *Exchequer*:

ONE OF

His MAJESTY'S

most Honorable Privy Council:

PRESIDENT

OF THE

ROYAL SOCIETY,

And one of the

MEMBERS of the

Honorable House of COMMONS,

THIS

TRACT

Is Humbly Dedicated by

Tho. Everard.

TO

The Right Honourable

CHARLES MONTAGUE, Esq;

One of the

Lords Commissioners

OF THE

TREASURY.

PROPOSALS

His PROPOSALS TO THE

most Honorable Privy Council:

FOR

PRINTING

BY

SUBSCRIPTION,

OF THE

ROYAL SOCIETY'S

PHILOSOPHICAL TRANSACTIONS.

IN the former Editions of this Work, I digested what materials I then met with, both concerning the Publick: and the Kind reception it found, encouraged me to publish this third Edition, for which I shall not at present make any Apo-

A 2

THE PREFACE TO THE READER.

IN the former Editions of this Tract, I signified what inducements I then had to make it publick: and the kind reception it then met with, hath incouraged me to publish this third Edition, for which I shall not at present make any Apo-

logy: for if this little Tract be in any measure serviceable to the end for which it was designed, viz. to instruct and assist the Officers of his Majesty's Excise, (especially Supervisors and Gaugers) in the ready and faithful discharge of their Duties; to describe and distinguish the various forms of Casks, and to recommend to their Practice proper Rules for finding the true Content of each; that so neither his Majesty, nor his Subjects may receive that wrong in the Gauging of Brandy, and other Exciseable Liquors, which one of them must necessarily be supposed to do; if the false Rules, which some late Writers have given, be frequently made use of. I say if this Tract be truly serviceable to this end,

it

it doth not (I conceive) need any Apology, and if it be not, I cannot make any that will be sufficient; and thus I might put an end to this Preface, but that I think it requisite to give a short account of the Additions I have made to this Impression.

As for instance, in Sect. VIII. I have laid down a Rule for Inching a Tun by Calculation.

In Sect. IX. I have endeavoured to describe and distinguish the various forms of Casks; and having noted the insufficiency of some of the common Rules, I have laid down others, by which the Content of any Cask may be readily found, either by the Pen or Instrument: here is also a Table of the Segments of a Circle,

with

with its Construction, and Use in finding the Ullage of a Cask, with two other Tables very useful for the same purpose.

In this Section you have also a Rule for finding the Diameters of the Frustum of a Spheroid, and its Content upon every Inch. Lastly,

In Sect. XI. I have shown the Use of the Instrument in the Resolution of several useful Questions relating to Interest both Simple and Compound. In the Rule of Fellowship. The proportion betwixt Lines and Solids, and in the measuring of Superficies and Solids; as Board, Paving, Cieling, Flooring, Tyling, Timber, Stone, Brick-work, &c. All which it performs with great expedition without the help of the Pen, and

and is therefore very useful for all Surveyors, or other Persons who are concerned in these Affairs.

In an Appendix to this Edition, I have endeavoured to explain the Nature and Original of several Solids; as Spheroids, Parabolick and Hyperbolick Conoids, Parabolick and Hyperbolick Spindles, and given variety of Rules for finding the Content of each, either the whole or the parts: a more full and clear account of these and several other Solids may be seen in the Writings * of the Learned Dr. Wallis, Professor of Geometry in the University of Oxford, (to whom I must acknowledge my self very much obliged.)

* De Sectionibus Conicis. Mechanica, Pars Secund. Treatise of Algebra.

obliged.) What is offered to this Appendix, and the Book it self, will I hope answer the end for which I intend it: I have not vanity enough to think it will please all; but the Candid and Ingenuous (and I value not the censure of others) will (I doubt not) afford it a favourable reception.

London,
Sept. 9. 1696.

T. F.

ADVER-

ADVERTISEMENT.

THE Instruments describ'd in this Book are made by *Isaac Carver*, at the Sign of the *Globe-Dial* in *Horsly-down*: who also maketh all other Mathematical Instruments in Silver, Brass, Ivory or Wood.

ADVERTISEMENT.

THE Instruments describ'd in this Book are made by *Isaac Carver*, at the Sign of the *Globe-Dial* in *Horsydown*: who also maketh all other Mathematical Instruments in Silver, Brass, Ivory or Wood.

THE INTRODUCTION OF Decimal Arithmetick.

WHoever will rightly understand the Art of *Gauging*, or the *Use* of the *Instrument* here describ'd, ought in some measure to be acquainted with the Art of *Arithmetick*; at least with *Numeration*, *Addition*, *Subtraction*, *Multiplication* and *Division*, both in *whole Numbers*, and *Decimal Fractions*; the latter of which (if well understood) is in this Art most useful.

And forasmuch as most of the *Problems* in this Tract are resolv'd by help of the *Line of Numbers*, the several *Uses* of which *Line* are render'd more easie by the knowledge of *Decimal Arithmetick*: I shall in the first place endeavour to shew what a *Decimal Fraction* is, and also give some Rules and Examples, by which any person who does at all understand the vulgar *Arithmetick*, may in a few hours time throughly comprehend this.

A *Decimal Fraction*, is that, which by prefixing a point or a prick towards the left hand, its value is decreased from so many Units, to so many Tenth

parts of an Unit, and if a point and a Cypher, or a Digit be prefixed, it will then be ſo many hundred parts, and if a point and two Cyphers or Digits be prefixed, its value is decreaſed to be but ſo many thouſand parts; as if you would prefix before the Figure 2, a point thus [.2] 'tis then decreaſed from 2 Units to 2 tenth parts of an Unit, and if you prefix a point and a Cypher thus, [.02] it is decreaſed from 2 Units to 2 hundred parts of an Unit. For,

As in *whole Numbers*, the value or denomination of places *increaſes* by Tens from the Units place towards the *left* hand, ſo in *Decimal Fractions*, the value or denomination of places *decreaſes* by Tens from the Units place towards the *right* hand, as in the following Scheme.

In which I have ſet an Unit in its due place, and all the Figures towards the left hand (being whole Numbers) do *increaſe* by Tens, for the Figure 2, next the Units place is 2 Tens, (or 20) the next is 3 hundreds, and the next 4 thouſands, *&c.*

Hundreds of Thouſands.	*Tens of Thouſands.*	*Thouſands.*	*Hundreds.*	*Tens.*	*Unit.*	*Tenth Parts.*	*Hundred Parts.*	*Thouſand Parts.*	*Ten Thouſand Parts.*	*Hundred Thouſ. Parts.*
6	5	4	3	2	1	2	3	4	5	6

But all the Figures from the Units place towards the right hand (being *Decimal Fractions*) do *decreaſe* by

BRITISH MUSEUM 30 60 NOV

by Tens, ſo the Figure 2, towards the right hand, is 2 tenth parts, the next is 3 hundred parts, and the next 4 thouſand parts of an Unit, each place towards the right hand being ten times leſs.

In *Decimal Arithmetick*, we always imagin (and it would be very commodious if it were really ſo) that all intire Units, Integers, and things are divided into Ten equal parts, and each of theſe parts ſo divided *we* call *Primes*, we alſo divide each of theſe *Primes* into other Ten equal parts, and every of theſe diviſions we call *ſeconds*, and each of theſe being divided into Ten other equal parts may be called *thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, ſo one Second is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ of an Unit, and one third is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{1000}$ part of an Unit) and ſo by dividing the former, and ſubdividing theſe latter, we may run on *ad Infinitum*.

Let a Pound Sterling be given to be Decimally divided:

According to the notion premiſed, the firſt diviſion muſt be *primes*, the next diviſion *ſeconds*, the next *thirds*, &c.

So if one Pound Sterling (being 20 *s.*) be divided into Ten equal parts, one of theſe parts is one *prime*, and its value is 2 *s.* and will ſtand thus, [.1] three *primes*, or three *tenths* of a Pound will ſtand thus, [.3] and its value is 6 *s.* Again, one *prime*, or .1, being divided into Ten equal parts, each of thoſe parts will be one *ſecond*, or one *hundred* part of a Pound, and is thus expreſſed, [.01] and its value will be 2 *d.* $\frac{1}{4}$ and $\frac{5}{10}$ of a Farthing; and if .01 be divided into Ten other equal parts, each of thoſe parts ſo divided will be *thirds*, or one *thouſand* part of a Pound, and will ſtand thus, [.001] and its value will be $\frac{24}{100}$ of a Farthing.

The like may be understood of one Pound *Troy* or *Averdupoize*, one Foot, one Gallon, or any other Integer or thing whatsoever.

A *Decimal* Fraction, whether it stand alone or be joyned with *whole* Numbers, is ever distinguished from a *whole* Number, by a point or a prick before it towards the left hand, as in these Examples.

.4752 4.752 47.52 475.2

In *Decimal* Fractions the *Numerators* only are set down, the *Denominators* being known by the *Number* of places in the *Numerator*, for if the *Numerator* consist of but one place, the *Denominator* is 10, if of 2 places the *Denominator* is 100, if of 3, 1000, if of 4. 10000, *&c.*

Example.

The *Denominator* of	.2 .25 .051 .0752	is	10 100 1000 10000

As Cyphers *before* a whole Number have no value, so Cyphers *after* a *Decimal* Fraction are of no signification, and therefore will not increase the Fraction nor alter the *Denominator*, for .2 is two tenth parts, and .20 is no more.

Again, as Cyphers *after* a whole Number do *increase* that Number; so Cyphers *before* a *Decimal* Fraction do *diminish* the value thereof, as in these *Examples.*

.5	is 5.	Tenth	Parts
.05		Hundred	
.005		Thousand	
.0005		Ten Thousand	

Here every Cypher added does remove the Fraction further from Unity, making it Ten times less than before.

Addition and Subtraction of Decimal Fractions.

1. AS for the operation of *Addition* and *Subtraction* in *Decimals*, it is the *very same* with the *vulgar*, there must only care be had of placing *Units* under *Units*, and *Fraction* under *Fraction*, in their proper Ranks and Files.

For Example.

To ——— 376.435
Add ——— 54.21

Sum is ——— 430.655

Again,

To ——— 26.51
Add ——— 527.342

Sum is ——— 553.852

Subtraction.

From	430.645
Subtract	54.21
Remains	376.435

Or, thus:

From	553.852
Subtract	527.342
Remains	26.510

Note, In *Addition* there muſt ever be as *many* places of Fractions in the *Total*, as are found in any of the Sums to be added together.

Multiplication of Decimals.

1. IN *Multiplication* both of *Decimal* Fractions and *mixt* Numbers, there are ever *two* Numbers given to find a *third* unknown.

One of the Numbers given (and it's no matter which) is called the *Multiplicator*, the other the *Multiplicand*, and the Number ſought is called the *Product*, and this doth ever contain one of the Numbers given, as many times as the other given Number does contain Unity.

Multiplication of *Decimals* is perform'd after the *ſame manner* as in *whole Numbers*, for the Numbers being ſet down one under another, we proceed in the Multiplication as if they were all whole Numbers, only

only it muſt be obſerved (when the operation is finiſh'd) how *many* places of *Decimal* Fractions are contain'd in the *Multiplicator* and *Multiplicand*, for *ſo many* muſt be in the *Product*, as in theſe Examples.

Multiplicand ——	35.25
Multiplicator ——	7.24
	14100
	7050
	24675
Product ——	255.2100

Or, Thus:

Multiplicand ——	6.75
Multiplicator ——	.25
	3375
	1350
Product ——	1.6875

2. When it happens (as it often does) that there are not ſo many *Figures* in the *Product*, as there are *Fractions* in the two Numbers to be Multiplied, you are then to place *Cyphers* before the *Product*, 'till the *Number* of places be equal, as in theſe Examples.

.04	.75	85
.6	.05	.005
.024	.0375	.425

3. As *whole Numbers* are increased by Multiplication, so *Decimal Fractions* are hereby made less, for the *Product* is removed farther from Unity than either of the Fractions given to be Multiplied, as appears by the last Examples.

4. When a *Decimal* Fraction or *mixt* Number is to be Multiplied by an Unit with Cyphers (as 10, 100, 1000, *&c.*) you need only remove the point so many places towards the right hand, as there are Cyphers with the Unit, thus if .7562 be Multiplied

By 10	The Product will be	7.562
By 100		75.62
By 1000		756.2
By 10000		7562

Division of Decimals.

1. IN *Division* there are *two* Numbers given to find a *third* unknown.

One of the Numbers given is called the *Divisor*, the other the *Dividend*, and the Number sought is called the *Quotient*, and this *Quotient* doth ever contain Unity as many times as the *Divisor* is contain'd in the *Dividend*.

Division of *Decimal* Fractions or *mixt* Numbers, is perform'd after the *same manner* as in *whole* Numbers, all the difficulty is to know the Denomination of places in the *Quotient*, and they are easily discovered by this one general Rule.

The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend, which at the first demand stands exactly over the place of Unity in the Divisor. I shall make this plain by some Examples. Example.

Example.

Let it be required to divide 32 *by* 51.2

In this Example the *Divisor* (51.2) is *greater* than (32) the *Dividend*, therefore in this and all the like cases you must place a competent Number of *Cyphers* behind the *Dividend*, which (if it be a whole Number, the) Cyphers so placed must be distinguished therefrom as Fractions, as here I put 4 Cyphers behind 32, and place the Numbers in this manner:

51.2) 32.0000 (

This done, I proceed to the division (as if all were whole Numbers) and first ask how many times the *Divisor* is contain'd in (320) the three first Figures of the *Dividend*, and seeing in this Example it is not contain'd in the three first Figures, it will therefore extend to the fourth place, over which I put a prick, to show that I must begin to Subtract at that place. I then ask how many times 5 in 32: I find 6, I place 6 in the *Quotient*, and Multiply the *Divisor* by 6, Subtracting the Product out of the *Dividend*, beginning my Subtraction at the place where I put the prick: Thus 6 times 2 is 12, 12 from 0 I cannot, I borrow 2 and say 12 from 20 and there remains 8, which I place under the Cypher: again, 6 times 1 is 6, and 2 I borrowed is 8; 8 from 10, and there rests 2, which I place under the next Cypher before 8, as in the Example; then 6 times 5 is 30, and 1 I borrowed is 31; 31 from 32, and there rests 1, which I set under 2; then for my new Dividend I bring down

51.2) 32.0000 (6
128

the next Cypher and set it behind the Remainder, and the Example will stand thus:

51.2) 32.0000 (6
1280

Then proceeding in my Division, I ask how many times 5 in 12. I find 2, which I place in the *Quotient*, and say 2 times 2 is 4; 4 from 10, and there remains 6, which I set under 0: again, 2 times 1 is 2, and 1 I borrowed is 3; 3 from 8, and there rests 5, which I place under 8, then 2 times 5 is 10; 10 from 12, and there rests 2, which I place under 2, and to this Remainder I bring down another Cypher to make my new *Dividend*, so will the Example stand thus:

51.2) 32.0000 (62
1280
256

51.2) 32.0000 (62
1280
2560

This done, I inquire how many times 5 in 25, and find 5, which I place in the *Quotient*, and say, 5 times 2 is 10, 10 from 10 and there rests 0; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6 and there rests 0; lastly 5 times 5 is 25, 25 from 25 and there remains 0; and thus my Division is finish'd, and the Example will stand thus:

51.2) 32.0000 (.625
1280
2560
0000

Now by the general Rule above given it is plain that the firſt Figure in the *Quotient* muſt be Tenths, and therefore the *Quotient* is all Decimal part, for the *Diviſor* being placed under the firſt Figures of the *Dividend*, this Example will ſtand thus:

32.0000 (.625
51.2

By which it appears, that the *Cypher* in the *Dividend*, which ſtands over the *Units place* in the *Diviſor*, is the place of *Tenths*;

See more Examples:

Diviſor.	*Dividend.*	*Quotient.*
51.2)	320.000	(6.25
512)	32.0000	(.0625
7.5)	.375	(.05

2. In Diviſion of *whole* Numbers or *mixt*, if there happen to be a Remainder, you may bring down more Cyphers, and by continuing your Diviſion, carry on the *Quotients* to as many places of Fractions as you pleaſe, four or five will be very near the truth, but two or three will be ſufficient in moſt Caſes, Example:

6.26) 25.800000 (4.12 14
760
1340
880
2510
36

3. When

3. When any Number (either *Decimal* or *mixt*) is to be divided by an Unit with Cyphers as 10, 100, 1000, *&c.* you need only *remove* the prick in the *Dividend*, so *many* places towards the left hand as there are *Cyphers* with the Unit, supplying the *vacant* places (if any be) with Cyphers.

Thus 756.2 divided by 10, is 75.62, Divided by a 100 it will be 7.562, by 1000, .7562, and if by a 10000 the *Quotient* will be .07562.

4. *Division* is *prov'd* by *Multiplication*; and *Multiplication* by *Division*.

To *prove Division*, Multiply the *Quotient* by the *Divisor*, and the *Product* is the *Dividend*: Thus, .652 (the *Quotient* in the first Example of *Division*) being Multiplied by 51.2 (the *Divisor*) the Product is 32.0000 (the *Dividend.*)

To *prove Multiplication*; divide the *Product* by *either* of the Numbers given to be Multiplied, and the *Quotient* will be the other Number given.

Thus if the last Product (*viz.*) 32.0000 be Divided by 51.2 the *Quotient* is .625, or if 32.000 be Divided by .625, the *Quotient* will be 51.2.

And hence 'tis evident, that *Division* is also *prov'd* by *Division*: for if any *Quotient* be made a *Divisor*, it will Quote the first *Divisor*, Example:

```
1.8) 28.8 (16
     108
     000

16) 28.8 (1.8
    128
    000
```

5. Now forasmuch as *Multiplication* is *easier* than *Division*; I shall here shew that what is perform'd

form'd by a *Diviſor*, may alſo be perform'd by a *Multiplicator*, and *how* ſuch *Multiplicator* may be found: Suppoſe I were to *Divide* 900 by 25, the *Quotient* will be 36.

Now let it be required to *Multiply* 900 by a certain Number that ſhall produce 36.

To *find* this *Multiplicator*, *Divide* an Unit with Cyphers by 25, the *Diviſor* propoſed, the *Quotient* will be .04 the *Multiplicator* ſought; for 900 being *Multiplied* by .04 the *Product* will be 36. which was required.

Again, Having a *Multiplicator* to find a *Diviſor*, this is but the *Converſe* of the former; for if an Unit with Cyphers be *Divided* by .04 (a *Multiplicator*) the *Quotient* will be 25, the *Diviſor* as before.

6. A *Vulgar* Fraction is reduced into a *Decimal* by *Diviſion*: for which the Rule is,

Divide the *Numerator* (of the *Vulgar* Fraction given) by the *Denominator*, and the *Quotient* will be a *Decimal* Fraction, *equal* in Value to the *Vulgar* Fraction given: So $\frac{3}{4}$ reduced as aforeſaid will be .75:

Example.

4) 3.00 (.75
20
00

Note, The *odd parts* of an Integer cannot be exactly reduced to a *Decimal*, for there will always be a *Remainder*, in ſuch Caſes carry on the Fraction to four or five places, it will be very near the truth:

Example.

Example.

$\frac{4}{9}$ 9) 40000 (.4444 &c.
40
40
40

7. To reduce a *Decimal Fraction* to the *known parts* of the *Integer*?

If the *Decimal* given be part of a *Pound* Sterling, Multiply it by 20 (the *Shillings* in a Pound, and the Fractions in the Product by 12, (the Pence in a Shilling,) and the Fractions in this Product by 4, (the *Farthings* in a Peny.) This done, the *whole* Numbers in the respective Products shew the *Shillings*, *Pence*, and *Farthings* contained in the Decimal given, and the *Fractions* in the last Product are Decimals of a Farthing. *Example*: Suppose .25 a Decimal Fraction of a Pound Sterling were given to be reduced, Multiply it by 20, the Number of Shillings in a Pound, the Product will be 5, *viz.* 5 *s.*

.25
20
Shillings ——— 5.00

So the *Value* of .60625 will be found to be 12 *s.* 1 $\frac{1}{2}$ *d.* as appears by the following Example:

.60625
20
Shillings ——— 12.12500
Shillings

Shillings —— 12.12500
12
25000
12500
Pence —— 1.50000
4
Farthings —— 2.00000

If the Decimal given be part of a *Beer Barrel*: Multiply it by 4, (the *Firkins* in a Barrel) and the Fractions in the Product by 9, (the *Gallons* in a Firkin,) and the Fractions in that Product by 8, (the *Pints* in a Gallon.) This done, the *whole* Numbers in the respective Products are equal to *Firkins*, *Gallons* and *Pints* contained in the Decimal propounded: Example.

Let .72 be a Decimal of a Beer Barrel to be reduced as aforesaid, it will be found to be 2 Firkins 7 Gallons 7 Pints and .36 of a Pint, as appears by the Work.

.72
4
Firkins —— 2.88
9
Gallons —— 7.92
8
Pints —— 7.36

But

But *these* and *many other* Arithmetical Questions may be more *readily resolved* by the *Instrument*, a description of which see in the first Chapter of the following Discourse.

ADVER-

ADVERTISEMENT.

IN the first Impression of this Tract, I gave the Explanation and Use of another Sliding-Rule of three Foot long, with one Sliding-Rod, that may be drawn out to six Foot; these with the Cane in which they are usually put, will extend to nine Foot in length, and are very useful in taking the Diameters of Tuns, and by reason of its length will resolve most Questions truer than the Foot Rule here described: but the Lines upon that being the same that are upon this, what is said of one may be understood of the other; this Foot Rule is sufficient for ordinary Practice. The Reader may use either, or both as he thinks fit.

STE-

ADVERTISEMENT.

STEREOMETRY,

OR, THE

ART of GAUGING

MADE EASIE

By the Help of a New

SLIDING-RULE.

SECT. I.

A Deſcription of the Rule, and of the ſeveral Lines upon it, with their Uſe in ſome Queſtions in Arithmetick.

THE Rule conſiſts of three Parts, *viz.* a Rule of 12 Inches long, and two ſmall Scales to ſlide in it, which may be drawn out, one towards the right hand, and the other towards the left, till the whole be three Foot long.

The principal Lines on the Inſtrument are thoſe commonly

commonly known by the name of *Gunters Line*, or *Line of Numbers*, which are here distinguish'd one from another by certain Letters, set at the end of the Lines towards the right hand.

Thus the Lines D are each of them one *single Line of Numbers*, beginning at the end of the Rule towards the left hand, nad from thence continued to the other end.

The Line A B and C are called *double Numbers*, each being *two* Lines or Radius's of Numbers, the *first* beginning at the *left* hand, and ending in the *middle* of the Rule, *where* the *second* Radius begins, and is from *thence* continued to the *end* at the *right* hand.

The Line E is called *Triple Numbers*, being *three* Radius's of Numbers; the *first* beginning at the *left* hand, and the *third* ending at the *right* hand.

This *Triple* Line is *equal* in length to the *double* Lines, and *all* to the *single* Line, for all the *five* begin and end at the *same* Point.

The *Line of Numbers* is (now) so *well known* to most persons, that it may be thought a *sufficient* description to have only said that *these* are *such*.

But for as much as this Instrument may be useful to some who (I presume) do not yet know what the *Line of Numbers* is, I shall therefore endeavour to explain it as followeth.

The *Line of Numbers*, is a Line of Geometrical proportions, divided first into *Nine unequal* parts call'd *Primes*, which are distinguish'd by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of these *Primes* are subdivided into *ten* other parts (according to the same Reason) called *tenths*, and again, each of those *tenths* are subdivided or at least suppos'd to be subdivided into *ten* other parts, according as the *length* of the Line will admit, as here the *Line* D, being about

11 In-

11 Inches long, each tenth in the first *Prime* is really subdivided into *ten* parts, call'd *Centesms*, but betwixt the Figures 2 and 4 each *tenth* is divided but into five parts, and therefore each of those parts do signifie *two Centesms*: again from the Figure 4 to the end of the Line every tenth is divided into two parts, each representing five *Centesms*: Lastly each of these *Centesms* is also supposed to be divided into *ten* parts, which by some are call'd *Millians*; but a Line of this length will not admit of this last division.

The Figures (1, 2, 3, 4, 5, 6, *&c.*) by which the *Primes* are distinguish'd, are all Arbitrary points, and may each of them represent so many intire *Units*. *Tens*, *Hundreds*, or *Thousands*; or they may also represent so many *Tenth*, *Hundredth*, *Thousandth*, or *Ten Thousandth parts* of an *Unit*.

1. For *whole* or *intire Units*: Let the first *Prime*, or the Fig. 1. at the beginning of the Line, D represent one *Unit*, then shall all the Figures towards the right hand (*viz.* 2, 3, 4, 5, *&c.* to 10.) represent *so many* Units, and the tenths in each *Prime*, will be *tenth parts*, and the *Centesms* in each of those tenths will be *hundred parts* of an Unit.

Or, let 1 at the beginning of the Line represent 10 Units, then will each *Prime* forwards represent 10 times so many Units as the Figures express; thus, the Figures 2, 3, 4, 5, *&c.* will be 20, 30, 40, 50, *&c.*

And when one Prime represents 10 Units, each tenth in that *Prime* will be 1, and each *Centesm* in these tenths will be 1 tenth part of an Unit.

Again, Let the first *Prime* represent 100, then the Figures 2, 3, 4, 5, *&c.* will represent 200, 300, 400, 500, *&c.* and therefore 10 at the end will be 1000, and according to this supposition 1 tenth in each *Prime* will be 10 Units, and in those tenths each *Centesm* will be 1.

2. For

2. For *Decimal Fractions*: Let 10 at the end of the Ling (at D) represent 1, then each *Prime* towards the left hand will be .1, and in those *Primes* each tenth will be .01, and in these tenths each *Centesm* will be .001 part of an Unit.

To make this more plain, draw out the sliding Piece B, till 1 at the *beginning* of the Line B, stand exactly against 10 at the *end* of the Line A, for then you have a Line of Numbers four times repeated; upon which let 1 at the *beginning* of the Line A, represent 1 Unit; then shall 1 in the *middle* of the said Line be 10. and 10 at the *end* thereof (or which is all one at the *beginning* of the Line B,) represent 100, and by consequence 1 in the *middle* of the Line B will be 1000, and for the *same* reason 10, at the *end* of the said Line (which is also the *end* of the *Rod*) will be 10000.

But keeping the Rule as it now stands, let 10 at the end of the fourth Radius (*viz.* at B.) represent 1, then shall each Prime in the said fourth Radius represent .1, in the third Radius .01, in the second Radius .001, and in the first .0001 part of an Unit.

So will 2 in the first Radius be .0002, in the second 002, in the third .02, and in the fourth .2 parts.

The *Numbers* and *Divisions* on the Lines being thus explained, it will not (I presume) be difficult to find the point upon the Line, where any Number given is represented.

As for example, Suppose the Number were 1895. for the first Figure thereof (*viz.* 1.) I count 1 at the beginning of the Line D, for the second Figure I count 8 tenths next following (that is 8 of the greater Divisions betwixt 1 and 2.) then from this point forward I count 9 *Centesms* for the third Figure, and for the last Figure 5 I count half the next *Centesm*; so I find the Point *ag* will represent 1895, and by

the

the ſame Rule the Number 1715 will be found at the Point *w g*; hence obſerve,

1. That (on a Line of this length) only the *four firſt* Figures of any Number propoſed can be diſcovered, for if the Number given were 189562, it would be repreſented at the ſame Point where the former 1895 was found (*viz.*) at *a g*.

2. That all Numbers which after the firſt Figure have nothing but *Cyphers* (as 20, 200, 2000, *&c.*) are all repreſented at the *ſame* Point.

So 20, 200, 2000, are all repreſented by the Figure 2, at the beginning of the ſecond *Prime*.

3. All Numbers conſiſting of *three* Figures, and having a Cypher in the *middle*, are found within the firſt tenth of that *Prime*, at which the firſt Figure of the Number given is found.

Example, Let the Number given be 308, for the *firſt* Figure I count 3 on the Line, (which I find at the beginning of the third *Prime*) now there being a Cypher in the *ſecond* place, I muſt not count any of the tenths, but for the *laſt* Figure 8, I count 8 *Centeſms*, and that is the Point which doth repreſent 308.

4. All Numbers conſiſting of *four* places, and having *two* Cyphers in the *middle*, muſt be ſought betwixt the beginning of the *Prime*, unto which they belong, and the firſt *Centeſm* of the ſame *Prime*; ſo 4005 being given, the *firſt* Figure (*viz.*) 4, is found at the beginning of the fourth *Prime*: Now there being Cyphers in the ſecond and third places, I muſt not count any of the tenths or *Centeſms*, but for the *laſt* Figure 5, I eſtimate 5 Millians, which is about the middle of the firſt *Centeſm*, and that is the Point where 4005 is repreſented.

Note, *Decimal* Fractions and *mixt* Numbers, are diſcovered after the *ſame* manner as whole Numbers

bers, for if the first Number 1895, were 18,95, it will be found at the *same point*, and by the *same rule* as the whole Number, (*viz.*) at *g*, and by what hath been said, it is easie to find what Number is represented at any Point upon the Line, as will appear in the following Questions of Proportion, of which I come now to treat.

In *Arithmetick* there are *three* several sorts of *Proportion*, *Arithmetical*, *Geometrical* and *Musical*.

Arithmetical when *divers* Numbers being *compared* together retain amongst themselves *equal* differences, as these, 2, 4, 6, 8, &c. And this is either *continued*, as in the Numbers before propounded, or in these, 3, 6. 9, 12, &c. (which is also called *Arithmetical progression*, or a *rank* of Numbers *Arithmetically* proportional;) or *discontinued*, as in these 2, 4. 8, 10, or the like.

Geometrical proportion, is when *divers* Numbers being *compared* together *differ* amongst themselves, according to the same rate or reason, as these 2, 4, 8, 16, &c. for here, as 2 is half 4, so 4 is half 8, and 8 half 16; this is likewise either *continued*, as in those before propounded, or in these 1,3,9,27,81, &c. (which are also call'd *Geometrical progression*, or a *rank* of Numbers *Geometrically* proportional;) or *discontinued*, as in these 2, 4, 16, 32, for as 4 is double 2, so is 32 double 16, but so is not 16 being compared with 4.

Musical proportion, being of no use in the present business, I shall not trouble the Reader with it: But if the two *former* (viz. *Arithmetical* and *Geometrical*) be *duly* consider'd, the following Problems may be the *better* understood.

Probl. I.

Problem I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

FInd *one* of the Numbers given upon the Line B, and ſet it againſt the *other* given Number on the Line A, then find the *ſame* Number upon B, (which was laſt counted upon A) and againſt it you have this *third* proportional upon A, and againſt this *third* upon B, is the *fourth* upon A; in like manner againſt the *fourth* upon B, you have the *fifth* on A, *&c.*

Example, Let it be required to find a *third* proportional to *theſe two* Numbers 2 and 4, which may bear the *ſame* proportion to 4, that 4 bears to 2.

Draw out the ſliding Rod till 2 upon B ſtand againſt 4 upon A, this done againſt 4 upon B is 8, the *third* proportional upon A, and againſt this *third* (*viz.* 8) upon B, is 16 upon A, which is the *fourth* proportional: Likewiſe againſt 16 upon B, is 32 the *fifth*, upon A, and againſt 32 upon B is 64 the *ſixth* proportional: but now proceeding forward, I find that 64 upon B, will reach *beyond* the end of the Line A, I therefore ſeek 64 towards the *left* hand upon B, and againſt it I find 128 the *ſeventh* proportional; and ſo proceeding farther you may find the *eighth* to be 256, the *ninth* 512, *&c. Contrariwiſe*, if it were required to find a *third* proportional to the *ſame* Numbers 2 and 4, which may bear the *ſame* proportion to 2, that 2 bears to 4.

Set 4 in the second Radius upon A, to 2 upon B, then against 2 upon A (towards the left hand) is 1 the *third* proportional, and against 1 upon A is .5 the *fourth* upon B, also against this *fourth* (*viz.* 5) upon A is .25, the *fifth* proportional on B, &c.

Multiplication by the Lines.

Problem II.

One Number being given to be Multiplied by another, to find the Product.

IN Multiplication either of whole Numbers, mixt, or decimal Fractions; the Proportion is:

As 1, *is to the Multiplicator*;
So is the Multiplicand, to the Product.

And the *Product* of any *two* Numbers shall have so *many* places as there be in *both* the Numbers given, except when the *lesser* of them do *not* exceed so *many* of the *first* Figures of the Product; then it will have *one* less.

1. Example: *Let it be required to Multiply* 6 *by* 4.

Say, [1 . 4 : : 6 . 24] which Analogy or Proportion may be read thus, *As* 1 is to 4, *So* is 6 to 24.

Therefore, Set 1 upon the Line B, to 4 on the Line A, and then against 6 upon B, is 24 upon A, which is the Product sought.

Note, the *Unit* or *first* term may be taken upon *either* of the Lines A or B, but the *first* and *third* terms must be counted upon *one* and the *same* Line, and

and the *second* on the *other* Line, where the *fourth* will also be found.

The Letters A and B may serve to distinguish the Lines.

2. Example: *Let the two Numbers given be* 68 *and* 26, *to find the Product.* The proportion is, [1 . 26 :: 68 . 1768.]

Therefore,

Set 1 upon B to 26 upon A, then against 68 upon B, is 1768 on A, which is the Product sought; Or,

Set 1 upon B, to 68 upon A, then against 26 upon B, is 1768 (on A) the Product. Therefore it matters not *which* of the Numbers given be made the *Multiplicator*; and *note* also, that the Product hath as many places as are in both the Numbers given, because the least of them (*viz.* 26) doth *exceed* so many of the first Figures of the Product, according to the Rule before given.

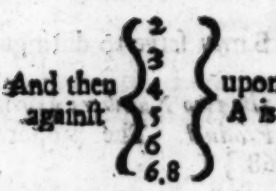
3. Example: *Let* 68 *be Multiplied by* 14. The proportion is,

1 : 14 :: 68 . 952

Therefore,

Set 1 upon B to 14 upon A; then against 68 upon B is 952 the Product upon A, and here the Product consists of one place *less* than there be in the two Numbers given, because the lesser of them (*viz.* 14) doth *not* exceed so many of the first Figures of the Product.

Now that the Product last found is 952, and not 95.2 nor 9.52 will thus appear: Set 1 upon B against 14 upon A,



By this 'tis evident, that the Lines are in effect a Table of Multiplication, for having set 1 to the Multiplicator, against any Multiplicand you have the Product; so if 2 be Multiplied by 14, the Product is 28; if 3, the Product will be 42; if 4, 56; if 5, 70, *&c.* Hence I conclude, that if the Multiplicand had been but 6.8, the Product would have been 95.2, but the Multiplicand being 68, the Product must be 952. for by taking away the prick, the Fractions in each are made whole Numbers.

When of *two* Numbers given to be Multiplied, the *one* consists of *whole* Numbers or *mixt*, and the *other* of *Fractions* only, make the *whole* or *mixt* Number the *Multiplicator*, and having set 1 against it, seek the Fraction towards the left hand, and against it you have the *Product*.

4. Example: *Let the two Numbers be* 27.5 *and* .8.

Set 1 on A, to 27.5 on B, and then against .8 (which being *less* than 1 I seek towards the left hand) on A is 22 the Product on B.

And notwithstanding a Number of *more* than four places cannot be exactly expressed on a Line of this length, yet the Product of any Multiplication may be discovered to *six* or *seven* places at least.

5. Example: *Suppose I were to Multiply* 2482 *by* 54. The proportion is,

1. 54 :: 2482 . 134028

There-

Therefore,

Set 1 upon A to 54 upon B, and then against 2 upon A, is 108 upon B: now suppose

$$2 \text{ be } \begin{Bmatrix} 20 \\ 200 \\ 2000 \end{Bmatrix} \quad 108 \text{ will be } \begin{Bmatrix} 1080 \\ 10800 \\ 108000 \end{Bmatrix}$$

So the Product will have six places, and against 2482 the Multiplicand, you may discern (upon the Line) the four first of them, (*viz.*) 1340, the two last may be found by Multiplying (in ones mind) the two last Figures of the *Multiplicand*, by the two last of the *Multiplicator*; for so shall you discover the two last Figures of the *Product*, which in this Example will be 28, which placed behind the four first (*viz.*) 1340, makes the Product complete 134028.

I might here add more Examples, but these already given may serve, they containing (I hope) sufficient directions for all the variety that can happen in Multiplication.

Division by the Lines.

Problem III.

One Number being given to be divided by another, to find the Quotient.

IN Division both of whole Numbers and mixt, the proportion is,

As the Divisor is to 1,
So is the Dividend to the Quotient.

Which Quotient shall ever consist of (but) so many Figures as the Dividend hath more than the Divisor, except when the Divisor does not exceed so many of the first Figures of the Dividend; then it shall have one place more.

1. Example: *Let it be required to divide* 24 *by* 4: The proportion is,

4 . 1 :: 24 . 6.

Therefore,

Set 4 upon B, to 1 upon A, and then against 24 upon B, is 6 upon A, which is the Quotient sought.

2. Example: *Let it be required to divide* 1768 *by* 26; The proportion is,

26 . 1 :: 1768 . 68

Therefore,

Set 26 upon B, to 1 upon A, and then against 1768 upon B, is 68 (the Quotient) upon A.

3. Example: *Suppose* 952 *were to be divided by* 14: The proportion is,

14 . 1 :: 952 . 68

Therefore,

Set 14 upon A, to 1 upon B, then against 952 upon A, you have 68 upon B, which is the Quotient required.

Observe here, the Dividend hath but one Figure more than the Divisor, yet the Quotient doth consist of two Figures, because the Divisor doth not exceed so many of the first Figures of the Dividend but in the first and second Examples the Quotients have but so many Figures as the Dividend hath more.

more than the Divisor, because the Divisor doth exceed so many of the first Figures of the Dividend, according to the general Rule above given.

This shews (in all Cases) how many Figures must be in the Quotient, the value of the first of which may be found by the Rule given in the Introduction, page 11.

Or it will thus appear, that the Quotient in the last Example is 68, and not 6.8, nor .68: Set 14 upon A, to 1 upon B,

And then against	28 42 56 70 84 95.2	upon A is	2 3 4 5 6 6.8	upon B.

This is but the Converse of the third Example of Multiplication, for as by that it is manifest that if 2 were Multiplied by 14, the Product would be 28; if 3, it would be 42; if 4, 56, *&c.* So here it is evident that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, *&c.* thus by reading on the proportion from the Divisor 14: 28, 56, 70, *&c.* I find at last, that if the Dividend were but 95. 2, the Quotient would be 6. 8, but the Dividend being 952, the Quotient must be 68, for by taking away the prick, the Fractions in each are made whole Numbers.

By these Examples it is also evident, that at *once* setting of the Rule we both *Multiply* and *Divide*.

For if 14 be a Multiplicator, set 1 on B, against 14 on A; this done, against any Multiplicand upon B, you have the Product upon A, as appear'd by the third Example of Multiplication.

And without moving the Rule, if you ſuppoſe 14 to be a Diviſor, then againſt any Dividend upon A, you have the Quotient upon B, as in the laſt Example of Diviſion.

How by *any Diviſor* to find a *Multiplicator*, was ſhewn in page 14 of the *Introduction*; It may alſo be performed by the *Lines* more readily.

Thus *having* a Diviſor to *find* a Multiplicator:

Set the Diviſor given upon A, to 1 upon B, and then againſt 1 (towards the left hand) upon A, is the Multiplicator upon B. Example, ſuppoſe 25 were a *Diviſor* given, to find a *Multiplicator*.

Set 25 upon A, to 1 upon B, and then againſt 1 (towards the left hand) upon A, is .04 the *Multiplicator* ſought.

By a *Multiplicator* to find a *Diviſor*: This is but the *Converſe* of the former, for having .04 a Multiplicator upon B, ſet it againſt 1 upon A, and then againſt 1 upon B, is 25 the Diviſor as before.

Problem IV.

To reduce a vulgar Fraction into a Decimal.

SET the Denominator (of the vulgar Fraction given) upon A, to the Numerator thereof upon B, and then againſt 1 towards the left hand upon B, is the Decimal Fraction ſought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25: For,

As 4 upon A is to 1 upon B; *So* is 1 upon A to .25 upon B:

Alſo $\frac{21}{28}$ will be .75, for *As* 28 upon A, is to 21 upon B; *So* is 1 upon A, to .75 upon B.

In

In like manner this Vulgar Fraction (*viz.*) $\frac{582}{960}$ is equal to this Decimal, *viz.* .60625, For, Set 960 upon A, to 582 upon B, then againſt 1 upon A, is .60625 upon B, and ſo for any other.

Problem V.

A Decimal Fraction being given, to reduce the ſame into the known parts of the Integer.

1. IF the Decimal be part of a Pound Sterling: Set 1 upon B, to the Number of Shillings, Pence or Farthings contained in a Pound upon A, then ſeek the Decimal given upon B, (towards the left hand) and againſt it you have the Shillings, Pence or Farthings (reſpectively) contain'd in the Decimal given.

Example: Suppoſe .7625 were to be reduced into Shillings.

Set 1 upon B to 20 (the Number of Shillings in a Pound) upon A, then againſt .7625 upon B, towards the left hand) is 15. 25, that is, 15 *s.* and .25 parts of a Shilling, which is equal to 3 *d.*

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then againſt .7625 towards the left hand upon B, is 183, the Pence contain'd in .7625.

3. But if the Decimal aforeſaid were to be reduced into Farthings.

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then again .7625 upon A, is 732, and ſo many Farthings are contain'd in .7625.

Again, ſuppoſe .7625 were a Decimal of an Ale Barrel to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons in a Barrel) upon B, then againſt .7625 upon A, is 24. 4, that is, 24

Gallons and .4 tenth parts of a Gallon; to reduce this .4 into Pints, ſet 1 upon A, to 8 upon B, and then againſt .4 upon A is 3.2 Pints: So .7625 of an Ale Barrel is equal to 24 Gallons 3 Pints and 2 tenth parts of a Pint.

Problem VI.

Three Numbers being given, to find a fourth in a direct proportion.

THis is call'd the *Rule of Three Direct*, and by the Inſtrument is wrought thus:

Set the firſt Number given upon B, to the ſecond upon A, and then againſt the third Number given upon B, is the fourth Number ſought.

Example: *If 8 Quarters of Mault will make 20 Barrels of Strong-Beer, How many Barrels of ſuch Beer will 22 Quarters make?*

Set 8 upon B, to 20 upon A, and then againſt 22 upon B, is 55 upon A, and ſo many Barrels will 22 Quarters make, and according to this proportion 24 will make 60 Barrels, 28 will make 70, alſo 32 Quarters will make 80 Barrels, &c.

Problem VII.

To three Numbers given, to find a fourth in an inverſed proportion.

THis is called the *Rule of Three Inverſe*, in which obſerve, that if the *third* Number be *greater* than the *firſt*, then will the *fourth* be *leſs* than the *ſecond*.

ſecond. And contrariwiſe, if the *third* Number be *leſs* than the *firſt*, the *fourth* will be *greater* than the *ſecond*, and in either Caſes the Rule is this:

Set the third Number upon A, to the firſt (being of like denomination) upon B, and then againſt the ſecond Number upon A, you have the fourth upon B.

Example: *If* 8 *Men do any piece of Work in* 9 *days, In how many days can* 12 *Men do the ſame Work?*

Set 12 upon A, to 8 upon B, then againſt 9 upon A, is 6 upon B, which is the anſwer. For 12 Men may do the ſame Work in 6 days, that 8 Men do in 9 days.

But if the Queſtion had been, *In how many days can* 6 *Men do the ſame Work?* the Anſwer will be 12: For,

As 6 upon B, is to 8 upon A, *So* is 9 upon B, to 12 upon A.

Problem VIII.

Betwixt two Numbers given, to find a mean Geometrically Proportional.

SET one of the Numbers given upon C, to the ſame Number upon D, and then againſt the other given Number upon C, is the Geometrical mean ſought.

Example: *Let the Numbers given be* 50 *and* 72, *to find a Geometrical mean*, &c.

Set

Set 50 upon C, to 50 upon D, and then against 72 upon C, is 60 upon D. So 60 is a Geometrical mean betwixt 50 and 72.

Or thus,

Set 72 upon C, to 72 upon D, and then against 50 upon C, is 60 upon D, the mean as before.

By the Pen thus:

Multiply one Number by the other, then Extract the Square-Root of the Product, this Square-Root is the Geometrical mean between the two Numbers given, thus 72 Multiplied by 50 is 3600, whose Square-Root is 60, the Geometrical mean between 50 and 72.

Problem IX.

To find the Square-Root of any Number under 1000000.

THE *Extraction of Roots* is one of the *hardest* Lessons in *Arithmetick*, yet by help of this Instrument it may be perform'd with *less trouble* than any of the foregoing Problems: For if the Lines C and D, be applied one to another, so as 10 at the end of D, be even with 10 at the end of C; I say the Lines thus applied are like a Table shewing the Square-Root of any Number by Inspection only; for against any Number upon C, you have the Square-Root thereof upon D, *Et Cont.*

Note, 1. When the Figures in the Number given are even, *viz.* when the Number consists of 2, 4, 6 or

or 8 Figures (being Integers) look the same in the second Radius of the Line C, and against it you have the Square-Root upon D: And in this case the said Root will ever consist of half as many Figures as the Number given.

Example: *Let 16 be the Number propounded,* I seek 16 in the second Radius upon C, and against it upon D, I find 4 the Square Root required.

In like manner	5.5	is the Root of	30.25	
	48		2304	
	886		784996	

2. When the Integers in the Number given are odd, *viz.* 1, 3, 5 or 7, seek it in the first Radius upon the Line C, and against it you have the Root sought. And in this case the Root will have half as many Figures as the Numbers given and one more.

Example: *Let the Number given be* 156.25, I seek this upon the first Radius of the Line C, and against it I find 12.5 the Root sought.

So also will	24	be the Root of	576
	144		20736
	1000		1000000

Problem X.

To find the Cube-Root of any Number under 1000000000.

PLace the Lines D and E one by another, so as 10 at the end of D, be even with 10 at the end of E,

E, this done againſt any Number upon E, you have the Cube-Root thereof upon D, *&c. Cont.*

Note, 1. When the Number given conſiſts of 1, 4 or 7 Figures (being Integers) find it in the firſt Radius of the Line E, and againſt it you have the Cube-Root ſought. Example: Let the Number given be 3375, I ſeek this in the firſt Radius in the Line E, and againſt it I find 15 upon D, which is the Cube-Root of 3375, and ſo is 212 the Cube-Root of 9528128.

2. When the Number given conſiſts of 2, 5 or 8 Integers, find it in the ſecond Radius upon E, and againſt it is the Root ſought. Example: Suppoſe 35.937 were propounded, find this in the ſecond Radius upon E, and againſt it is 3.3 the Cube-Root upon D; in like manner is 275 the Cube-Root of 20796875.

3. When the Number given conſiſts of 3, 6 or 9 Integers, it muſt be ſought in the third Radius, *&c.* For againſt it is the Cube-Root: Thus againſt 125 in the third Radius upon E, I find 5 the Cube-Root, and ſo likewiſe is 888 the Cube-Root 700227072.

Laſtly, to know how many places of Integers muſt be in the Cube-Root of any Number given.

Put a point over the place of Units in the Number given, then omitting 2, point every third Figure toward the left hand, then tell how many points, for ſo many places of Integers muſt the Cube-Root conſiſt of.

SECT.

Sect. II.

Of a Superficies.

1. A *Superficies* is a Figure incompaſſed about with a Line or Lines, and is either Round or Angular.

2. A *Round Figure* is that which is contain'd by one Round Line, and is either a Circle (as Fig. 1.) or an Ellipſis (as Fig. 2.)

3. An *Angular Figure* is that which doth conſiſt of three or more Angles, from the Number of which they are denominated, as a Figure of three Angles is called a Triangle, of four a Quadrangle, *&c.*

4. A *Triangle* is a Superficies comprehended by three Right-Lines (as Fig. 3.) of theſe there are ſix varieties, which I ſhall forbear to mention, they being all meaſured by one and the ſame Rule.

5. A *Quadrangle* is a Figure comprehended by four Right-Lines, and is either a *Parallelogram* or *Trapezium*.

6. A *Parallelogram* is a Figure whoſe oppoſite ſides are parallel, having equal diſtances from one another in all places, and is either Right or Oblique.

7. A *Right-Angled Parallelogram*, is that whoſe Angles are all right, and is either a Square (as Fig. 5.) or an Oblong (as Fig. 4.)

8. The *Oblique-Angled Parallelogram*, is that whoſe Angles are all Oblique, and is either a Rhombus (as Fig. 7.) or a Rhomboides (as Fig. 6.)

9. A *Trapezium*, is a Quadrangular Figure, whoſe four ſides are not all equal, (as Fig. 8.

10. *Figures*

10. *Figures* consisting of *more* sides than *four* are almost *innumerable*, but are reducible unto *two* sorts, *Regular* or *Irregular*, either of which are also call'd *Polygons*.

11. *Regular Polygons* are such whose sides and Angles are equal, they take their names from the Number of their sides, as that of five sides is call'd a *Pentagon*, (as Fig. 9.) that of six sides an *Hexagon*, &c. Of *Irregular Polygons*, 'tis needless to say any thing, they being measur'd after the same manner as a *Trapezium*.

I should next proceed to shew the Use of the Rule in the *measuring* of a Superficies, but it will not be altogether impertinent if I first premise;

That every *Magnitude* must be measur'd by some *known* kind of Magnitude that is *Homogeneal* (or like) to it. A *Line* is measur'd by a *Line*, as one Lineal Inch, Foot or Yard.

A *Superficies* is measur'd by a *Superficies*, as one Square Inch or Foot, &c. A *Solid* is measur'd by a *Solid*, as one Cubick Inch, one Cubick Foot, &c.

And when it is known

How many	Lineal	Inches or Feet are contained in a	Line.
	Square		Superficies.
	Cubical		Solid.

Then is the Quantity or Content of either of these kind of Magnitudes said to be known.

And the measures of *Capacity* commonly used in *England* are of two sorts, either *wet* or *dry*, for measuring of *Liquids*, as Beer, Syder, Wine, &c. (we take it for granted that) there are *two* distinct Gallons

Gallons and a *third* for *dry* Commodities, as Corn, &c. the Gallon for Beer and Ale, contains 282, for Wine 231, and for Corn 272 Solid Inches.

See these more fully explain'd in the following Tables, each of which doth shew how many Solid Inches are contained in any of the other Measures expressed therein.

A Table of Beer Measure.

Inches.					
$25\frac{1}{4}$	*Pints.*				
$70\frac{1}{2}$	2	*Quarts.*			
282	8	4	*Gallons.*		
2538	72	36	9	*Firkins.*	
5076	144	72	18	2	*Kilderkins.*
10152	288	144	36	4	2 *Barrel.*

A Table of Ale Measure.

Inches.					
$25\frac{1}{4}$	*Pints.*				
$70\frac{1}{2}$	2	*Quarts.*			
282	8	4	*Gallons.*		
2256	64	32	8	*Firkins.*	
4512	128	64	16	2	*Kilderkins.*
9024	156	128	32	4	2 *Barrel.*

By an Act of the First of *William* and *Mary*, 34 Gallons is the Barrel, both for Beer and Ale, in all places, except within the Weekly Bills of Mortality.

A Table of Wine Measure.

Inches.							
$28\frac{1}{8}$	*Pint.*						
$57\frac{1}{4}$	2	*Quart.*					
231	8	4	*Gallon.*				
4158	144	72	18	*Rundlet.*			
14553	504	252	63	$3\frac{1}{2}$	*Hogshead.*		
19404	672	336	84	$4\frac{2}{3}$	$1\frac{1}{3}$	*Tercion.*	
29106	1008	504	126	7	2	$1\frac{1}{2}$	*Pipe.*
58212	2016	1008	252	14	4	3	2 *Tun.*

This Table shews that in 1 *Tun* there is 2 *Pipes*, 3 *Tercions*, 4 *Hogsheads*, 14 *Rundlets*, 252 *Gallons*, 1008 *Quarts*, 2016 *Pints*, and 58212 Solid *Inches*. The Tables for *Beer* and *Ale* are like this, and need no explanation.

SECT.

Sect. III.

The Use of the Rule in Measuring of Superficies, and first of a Circle.

THE Area or Superficial Content of a Circle is found by knowledge of the Diameter or Circumference, I shall therefore first shew how by either of these to find the other.

Problem I.

The Diameter or Circumference of a Circle either being given to find the other.

THE Circumference of that Circle whose Diameter is Unity (or 1) is *3.14159265363; but for our purpose 3.141592, will suffice: Therefore, as 1 is to 3.141592, so is the Diameter of any Circle to the Circumference: By the Instrument thus,

*Foster's *Problemata Geomet. Varia. Propos.* 2.

Set 1 on the Line A, against 3.141592 on the Line B, this done against any Diameter on the Line A, you have the Circumference on the Line B, and the contrary thus, Against,

These

These Diameters.	20	you have these Circumferences.	62.831
	30		94.247
	40		125.663
	50		157.079

Or Contrariwise against,

These Circumferences.	20	you have these Diametera.	6.366
	30		9.549
	40		12.732
	50		15.915

And so of any other.

Problem II.

The Diameter of any Circle being given to find the Area (or any part thereof) in Inches, in Ale or Wine Gallons, and also in Ale or Beer Barrels.

1. *For the whole Area in Inches.*

THE Area of a Circle is equal to the Fact or Rect-Angle of half the Diameter into half the Circumference, that is to say, if half the Diameter be Multiplied by half the Circumference, the Product will be the Area.

Thus, when the Diameter is 1, the Circumference is 3.141592, the half of this is 1,570796, which Multiplied by half the Diameter, (*viz.* .5) the Product will be the Area of that Circle whose Diameter is 1, (*viz.*) .785398.

The

The Areas of all Circles are in proportion one to another, as the Squares of their Diameters, (2. 12. of *Euclid.*)

Therefore, as the Square of the Diameter of any Circle is to the Area of that Circle;

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above mentioned the Diameter is 1, and the Area .785398: Now the Square of 1 being but 1 it muſt hold: *As* 1 is to .785398, *So* is the Square of any Diameter to the Area: So .785398 is a fixed Multiplicator, and if an Unit with Cyphers be divided by .785398 the Quotient will be 1.27324 a fixed Diviſor, and by either of theſe fixed Numbers the Area of any Circle may be found, either by Multiplication or Diviſion, if the Diameter be firſt given. For if the Square of any Diameter be

Multiplied	by	.785398	Prod.
Divided	by	1.27324	Quot.

the Area in Square Inches, Feet or Yards, according as the Diameter was meaſured by Inches, *&c.*

But with more expedition by the Inſtrument, Thus,

Set 1 (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

This done the Lines are like a Table of Circles Areas to all Diameters, for againſt any Diameter upon the Line D, you have the Area thereof upon C.

Example, *Let the Diameter be* 20.

Set 1 upon D, to .785398 upon C, and then againſt 20 upon D, is 314.159 the Area required upon C, and as the Rule now ſtands I alſo find that when

The

The Diameter is	25 30 40	the Area will be	490.87 706.85 1256,63	Inches.

Contrariwise when the

Area is	300 400 500	the Diameter will be	19,54 22,56 25.23	

2 *For the Area in Gallons or Barrels.*

The Area in Inches Divided by 282 or 231, Quote the Area in Ale or Wine Gallons respectively, and so for any other Measure expressed in the former Tables, but without knowing the Area in Inches, the Area in Gallons may be found thus: Divide .785398

By	282 231	the Quotient is	.002785 1 .0033999	A G W G

The several Quotients are the Area of a Circle whose Diameter is 1, in Ale or Wine Gallons, respectively, and are fixed Multiplicators for finding the Area of all Circles in either of the Measures, for if the Square of the Diameter of any Circle be Multiplied by either of these Numbers, the Product is the Area in Ale or Wine Gallons, respectively.

If you would effect this by Division, the several Divisors are thus found, Multiply the Divisor for finding the Area in Inches, *viz.* 1.27324

By	282 231	the Product is	359.05 294.11	A G W G

These

These several Products are the Divisors sought. And the Square of the Diameter of any Circle divided by one of these, quotes the respective Area.

Having thus found the several fixed *Multiplicators* and *Divisors* for finding the Area of a Circle in Inches, Gallons and Barrels, I shall here represent them altogether as in the following Scheme.

	Multiplicators.	Divisors.
Inches.	.785398	1.27324
Ale Gallons	.0027851	359.054
Wine Gallons.	.0033999	294.118
Ale Barrels.	.00008703	11489.71.
Beer Barrels.	.00007736	12925.93

Thus by the Pen.

But the Area of any Circle may be more readily found by the help of certain fixt Numbers called Gage-Points, and these fixt Numbers are the Diameters of those Circles whose Content at one Inch deep is equal to the respective Gallons to which they belong, thus the Gage-Point for the Ale-Gallon is 18.95 which is the the Diameter of that Circle, whose Content is 282 the Square Inches in the Ale-Gallon.

The several Gage-Points are the Square-Roots of the Divisors last mentioned, and by the Rule are all found at once by the Lines C and D, for (setting these Lines even at the end)

Against { 359.05 } upon C. are these { 18.95 } Gage-Points upon D. { 18.95 } A G
these Divis. { 294.11 } { 17.15 } W G

To which are set the Letters *a g.* and *w g.* upon the Rule. Now

Now by these Gage-Points the Area of any Circle may be thus found.

For Ale-Gallons.

Set 18.95 (the Gage Point for Ale-Gallon) upon D, to 1 upon C, then against any Diameter upon D you have the Area upon G.

So if the Diameter were 40 Inches, the Area will be 4.45 Ale-Gallons, and the Rule being thus set, the Lines are like a Table of Circles Areas, for I likewise find that if the Diameter

Be	45 50 52.5 58.6	the Area will be	5.64 6.96 7.67 9.56	A G

What hath been said of finding the Area of a Circle in Ale-Gallons, may be understood of Wine Gallons, and also of Ale or Beer Barrels, observing to use the proper Gage-Points.

Note, When the Area of any Circle is sought in Ale or Wine-Gallons, if the Diameter be more than 18.95 yet less than 100, set the Gage-Point upon D, to 1 at the beginning of C, then against any Diameter from the Gage Point to 100 upon D, you have the Area upon C. Thus the Rule being set for Ale, you will find that when the

Diamet. is	30 40 60 80 100	the Area will be	2.507 4.456 10.026 17.825 27.851	Gallons.

When

When the Diameter is leſs than the Gage-Point or more than 100, then ſet the Gage-Point to 1 in the middle upon C, this done againſt theſe

Diame-	15	you have	.627	
ters up-	13	theſe Area's	.471	A G
on D	10	upon C	.278	

And without moving the Rule, if 10 at the beginning of D be 100, the Area againſt it will be 27.851: So againſt this Diameter, *viz.* 200, the Area is 111.4 Gallons, againſt 300, it is 250.7, and ſo on to 600 Inches Diameter, againſt which you have 1002.9, *&c.* by this you may further obſerve, that if the Diameter be increaſed by Tens, the Area will increaſe by Hundreds: thus,

the Diameter	10	gives	.27851
	100		27.851
	60		10.026
	600		1002.6 Gallons.

The like for Wine-Gallons.

Laſtly,

To find any part of the Area of a Circle in Ale or Wine-Gallons, &c.

Set the Gage-Point to $\frac{1}{3}$, $\frac{1}{2}$ or any other part of 1; then againſt the Diameter you have the like part of the Area.

Example: *Let it be required to find the third part of the Area of a Circle in Ale-Gallons.*

Set the Gage-Point to $\frac{1}{3}$ of 1, *viz.* .333, then against any Diameter you have $\frac{1}{3}$ of the Area, thus against 100 is 9.283, which is $\frac{1}{3}$ of 27.85. the whole Area.

Understand the like for Wine-Gallons, Ale or Beer Barrels.

Problem III. Figure 2.

The two Diameters c d *and* e f *of an Ellipsis, being given to find the Area or Content in Ale Gallons.*

AS the Square of the Diameter of a Circle, is to the Area of that Circle; So is the Rect-Angle or Product of the greater and lesser Diameters of an Ellipsis to the Area thereof. Therefore Multiply the greater Diameter by the lesser Diameter, then that Product Multiplied or Divided by the fixed Multiplicators or Divisors given (in pages 46 and 47,) gives the Area in Inches, Gallons or Barrels, according to the Number made use of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical mean Proportion between the greater and lesser Diameters, for this mean is the Diameter of a Circle whose Area is equal to the Area of the Ellipsis.

Example. Fig. 2.

Let the greater Diameter *c d* be 72 Inches, and the lesser *e f* 50, by the Rule above cited, the Geometrical mean between them will be found to be 60, the Diameter of a Circle equal to the Ellipsis, and the Area of a Circle, whose Diameter is 60, will be found

found to be 10.02 Ale-Gallons But the Area of an Ellipſis may be more eaſily found by the Rule, thus;

Set 359.05 upon B to one of the Diameters (ſuppoſe 50) upon A; then againſt the other Diameter (ſuppoſe 72) upon B, you have the Area upon A, which in this Example will be 10.02, Ale-Gallons, the Content of this Ellipſis at one Inch deep: the like may be done for Wine-Gallons, Ale or Beer Barrels by taking the reſpective Numbers laid down in page 47.

Problem IV.

To find the Area or Content of a Triangular Superficies.

IN all Right-lined Triangles, Multiply half the longeſt ſide (or Baſe) by the Perpendicular (which is always the neareſt diſtance from the Baſe to the oppoſite Angle;) or Multiply half the Perpendicular by the whole Baſe; this done, the Product is the Area required.

Example.

Let Fig. 3. repreſent a Triangular Back or Cooler, whoſe longeſt ſide *a b*, is 260 Inches, and the Perpendicular Line *c o* 110 Inches, the half of *a b* is 130, this Multiplied by 110 gives 14300 for the Area in Square Inches, and this divided by 282 quotes 50.7 the Area in Ale-Gallons, or by the Rule at one operation thus:

Set 282 upon B to 130 upon A, then againſt 110 upon B, is 50.7 upon A, the Content as before.

To meaſure Quadrangular Figures.

Problem V.

In any Right-angled Parallelogram be it Square or Oblong: The Product of any two ſides including one and the ſame Angle, is equal to the Area or Content.

1. Example.

IN (Fig. 5.) the ſides *h i* and *i k* are equal, ſuppoſe each be 138 Inches; this Multiplied by it ſelf is 19044 the Area in Square Inches: For the Area in Gallons by the Rule:

Set 282 upon B, to 138 upon A, then againſt 138 upon B, is 67.53 the Area in Ale-Gallons at one Inch deep.

2. Example. *Of an Oblong* (*Fig.* 4.)

Suppoſe *n s* 130 Inches, and *s u* 180, the Product of theſe (being Multiplied) is 23400 the Area in Inches: but for Ale Gallons, ſet 282 upon B, to 180 upon A, then againſt 130 upon B, is 82.9 Ale Gallons the Area required.

Problem

Problem VI.

In any Oblique Angled Parallelogram (be it a Rhombus or a Rhomboides) Multiply the ſhorteſt diſtance between the two longeſt ſides by one of the ſaid ſides, the Product is the Area.

1. Example. *Of a Rhombus* (*Fig.* 7.)

Suppoſe the ſide *p t* were 130 Inches, and the diſtance *q y* 108, theſe Multiplied the Product will be 14040 the Area in Inches: But by the Rule thus,

As 282 is to 108, *So* is 130 to 49.78, the Area in Ale-Gallons at one Inch deep.

2. Example. *Of a Rhomboides* (*Fig.* 6.)

Let the ſide *x z* be 260, and the diſtance *t w* 108, the Product of theſe is 28080 the Area in Inches. By the Rule:

Set 282 upon A, to 260 upon B, then againſt 108 upon A, is 99.57 the Area in Ale-Gallons.

Problem VII. Figure 8.

To find the Area or Content of a Trapezium.

A Right Line drawn from one of the Acute Angles to the Angle oppoſite, (as the Line *a b*) will divide the Trapezium into two Triangles, the Area's of which are equal to the whole Trapezium, and may be found by the 4. Problem of this Section: Or thus, Multiply the ſum of the two Perpendiculars (*ſ e* and *o r*) by the half of *a b*, the Product is the Area of the Trapezium, this is ſo plain that it needs no Example.

All other irregular Right-lined Figures, conſiſting of more than four ſides, muſt be divided into Triangles, (which will ever be leſs by two than the Number of ſides) and then the Area's of all thoſe Triangles are equal to the Area of the whole Figure.

Problem VIII.

The ſides of any Regular Polygon being given to find the Area.

IN any Regular Polygon, Multiply half the Sum of the ſides by the Perpendicular (or neareſt diſtance from the Center to one of the ſides) this done the Product is the Area.

Example, in a Pentagon (or Figure of five equal ſides;) as Fig. 9.

Suppoſe each ſide be one Inch, to find the Perpendicular (C A) we have given A B = .5, and the Angle at C = 36 *d.*

Therefore,

Therefore,

		Log.
As the *sine* C = 36 *d.*	*Co. Ar.*	0.230781
Is to the side A B = .5		0.698970
So is the *sine* B = 54. *d.*		9.907957
To the side C A = .68819		.837708

which is the Perpendicular sought, and this .68819 Multiplied by half the sum of the sides, *viz.* 2. 5, the Product is 1.72047, the Area of a Pentagon whose side is 1 Inch; and by this method I find the Area of all the other Polygons which are expresst in the following Table. In the first Column of which you have the names of the Polygons, in the second their Area's in Inches, each of which being divided by 282 quotes the Area in Ale-Gallons, which are the Numbers in the third Column.

Names of Polygons.	*Areas in Inches.*	*Areas in Ale-Gall.*
Pentagon	1.72047	.006101
Hexagon	2.59809	.009212
Heptagon	3.63440	.012838
Octagon	4.82840	.017120
Nonogon	6.18210	.021920
Decagon	7.69400	.027280
Undecagon	9.36760	.033210
Dodecagon	11.1960	.039700

Now having the Area (both in Inches and Ale-Gallons) of these Polygons when the side is Unity,

the Area of each may be readily found when the side is any known length: For,

As the Square of the side of any Polygon is to the Area of that Polygon;

So is the Square of the side of any other like Polygon to the Area thereof:

Therefore,

If the Area of any Polygon (expressed in the Table) be desired.

Multiply the Square of the side by a Number in the second Column against the name of the Polygon given, the Product will be the Area in Square Inches.

And if the Square of the side taken in Inches be Multiplied by a Number in the third Column, the Product is the Area in Ale Gallons.

Example.

Suppose the side of a Pentagon be 50 Inches, this Squared is 2500, which Multiplied by the first Number in the second Column, (*viz.*) 1.72047, the Product will be 4301.175, the Area in Square Inches.

And if the Square of 50 (*viz.*) 2500 be Multiplied by the first Number in the third Column, *viz.* .006101, the Product will be 15.252 the Area in Ale-Gallons, which agrees exactly with the former, for if you divide the Area in Inches, *viz.* 4301.175 by 282, the Quotient will be 15.252,

This may be easily performed by the Instrument, for there you have the Square of any Number by Inspection and the Multiplication is quickly wrought by the Lines A and B.

But

But the Lines C and D, will effect the whole with much more expedition, for at once setting of the Rule you have the Area (in Inches or Ale-Gallons) to any given side, or the side to any Area given.

Example.

For the Area in Ale-Gallons, Set 1 upon D to (the Area in Gallons when the side is 1, *viz.*) .006101 upon C, this done, against any side upon D you have the Area upon C, and the contrary.

Thus against 50 upon D, is 15.252, the Area in Ale-Gallons: Also against 22 Gallons an Area upon C, you have 69 the side upon D, *&c.*

It is needless to give Examples of the other Polygons, for what hath been said of the Pentagon may be understood of the rest, remembring to use the proper Numbers as aforesaid.

SECT. IV.

The Use of the Rule in the Mensuration of Solids, and first of a Prism.

A *Prism* is a Solid contained by several Planes, two of which being opposite are called the Bases, and these are equal, Parallel, alike and alike situate, but the other Planes are Parallelograms, in which a Right Line may be every where applied from Base to Base: Under this name *Prism* is comprehended that Solid of two Circular Bases, usually called a Cylinder.

Problem I.

To find the Solid Content of a Prism.

MUltiply the Area of the Base by the Perpendicular height, the Product is the Content sought.

Example.

Let (Fig. 11.) *represent a Brewers Tun in the form of a Square Prism, whose Base* g, h, k, i, *is equal to Fig.* 5. *each side being* 138 *Inches, and the Perpendicular* (n i) 30 *Inches, how many Ale-Gallons or Beer Barrels will this Tun contain?*

Set 282 upon B to 138 upon A, then against 138 upon B is 67.534 upon A, the Content in Ale-Gallons at one Inch deep, this Multiplied by 30, the whole depth gives 2026.02 the whole content in Ale-Gallons, this divided by 36 quotes 56.283 the Content in Beer Barrels.

The Fraction .283 (being reduced) is 10 Gallons and .188 parts. For

As 1.000 : 36 :: .283 : 10.188.

2. Example. *Of a Round Prism or Cylinder.*

Let Fig. 12. Represent a round Tun, whose Diameter (n o) at top is equal to (s y) the Diameter at bottom, each being 120 Inches, and the Altitude e x 36 Inches, How many Gallons or Beer Barrels may this Tun contain? First for Gallons.

Set

Set the Gage-Point (*a g*) to 36 the Tuns Depth upon C, then againſt 120 the Diameter upon D, is 1443.6 the Content in Ale-Gallons.

2. For the Content in Beer Barrels: 1443.6 Divided by 36, quotes 40.1 the Content in Barrels.

Suppoſe I come to this Tun, and find the Liquor 9 Inches deep, How many Beer Barrels are then contained in the Tun?

Set the Gage-Point to 9, then againſt 120 is 10.02, that is, 10 Barrels and 1 Gallon *fere*, the quantity of Liquor ſought.

Problem II.

The Diameter and depth in Inches, and the Content in Ale-Gallons of any Cylindrical Tun, any two being given the third may be found.

IN this Problem are Three Queſtions, but all reſolved at once ſetting the Rule.

1. By the depth and Content to find the Diameter.

Example.

Suppoſe the depth 40 Inches, and the Content 1800 Ale-Gallons. What is the Diameter?

Set 40 the depth upon C, to the Gage-Point upon D: Then againg 1800 the Content upon C is 127.1 Inches, the Diameter ſought.

2. By

2. By the Diameter and Content to find the depth, without moving the Rule: Say,

As 127.1 the Diameter is to 1800 the Content.
So is the Gage-Point to 40 the depth } upon { D C D C }

3. By the Depth and Diameter to find the Conten, the Rule ſtanding as before: Say,

As the Gage-Point is to 40 the depth.
So is 127.1 the Diameter to 1800 the Content } upon { D C D C }

Note, As a Circle is the Baſe of a Cylinder or round Priſm, ſo a Triangle, Quadrangle, or any other plain Superficies may repreſent the Baſe of a Solid, for if there be Planes erected perpendicularly upon the Line or Lines, which incompaſs any ſuch Superficies, they will generate a Solid which may be called a Priſm, and the Content of any ſuch Solid is gotten by Multiplying the Area of the Baſe by the Altitude or diſtance from one Baſe to another.

SECT.

Sect. V.

The Use of the Rule in the Mensuration of a Sphere, and its Fruſtums.

Problem I.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons, both Ale and Wine.

A Sphere is two third parts of a Cylinder, whoſe Diameter and Altitude are equal to the Diameter of the Sphere;

But if the Diameter and Altitude of a Cylinder be 1, the Area of the Baſe (which is alſo the Solid Content (for 1 doth not Multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, *viz.* .523598 is the Content of a Sphere whoſe Diameter is Unity or 1.

Morrover, all Bodies are in proportion one to another as the Cube of their like ſides:

Therefore,

As the Cube of the Diameter of any Sphere is to the Content of that Sphere;

So is the Cube of the Diameter of any other Sphere to the Content thereof.

Now if the Diameter be 1, the Cube thereof is but 1.

Therefore,

Therefore,

As 1 is to .523598:
So the Cube of the Diameter of any Sphere to the Content.

Example. *Let the Diameter of a Sphere be* 20 *Inches, and the Content required in Solid Inches.*

By the Lines D and C on the Rule, the Cube of 20 will appear to be 8000, this Multiplied by .523598, is 4188.78 the Content sought.

For the Content in Gallons.

If .523598 be divided

By { 282 / 231 } quotes { .001856 / .002266 } the content in { A. G. / W. G.

Now if the Cube of the Diameter of any Sphere be Multiplied by either of these Numbers, the Product will be the Content in Ale or Wine Gallons respectively: Thus, The Cube of 20 the Diameter (*viz.*) 8000 Multiplied by .001856 is 14.848 the Content in Ale-Gallons, and the like may be done for Wine.

But the Content of any Sphere may be more readily found by the Instrument.

Thus,

Set 1 upon D, to .523598 upon E, then against any Diameter upon D, is the Content upon E: So against 20 (the Diameter above-mentioned) upon D, is 4188.78 the Content in Inches as before: In like manner

manner if the Diameter were 30, the Content will be 14137.14, &c.

For Ale-Gallons thus,

Set 1 upon D. to .001856 upon E, then against any Diameter upon D, is the Content upon E, as against 20 upon D is 14.84, the Content as before, and without moving the Rule I find that if the

Diameter be	30	the content is	50.13	A. G.
	40		118.78	

Problem II.

Having the Altitude of the Fruſtum of a Globe, together with the Diameter of its Baſe, to find the Altitude of the other Fruſtum or (which is all one) the remainder of the Globes Axis.

The Rule is,

DIvide the Square of the Semidiameter of the Fruſtums Baſe, by the Altitude of either of the Fruſtums, the Quotient will be the Altitude of the other Fruſtum.

Example.

In Fig. 10. Let *c b* the Altitude of the Fruſtum be 6 Inches, and *d c e* the Diameter at the Baſe 24, the half of this (*viz. d c*) is 12, which Squared is 144, this Divided by the Altitude *b c*, *viz.* 6, the Quotient

Quotient is 24 equal to *c a*, the Altitude of the other Fruſtum. Note alſo, that a Geometrical mean proportion between *a c* and *c b*, is equal to *c d*, which doubled is *d e*, the Diameter of the Fruſtums Baſe.

Problem III.

Having the Altitude of the greater and leſſer Fruſtums of a Globe, and the Diameter of the Baſe, to find the Content of the leſſer Fruſtum.

The Rule.

1. BY the Diameter at the Fruſtums Baſe find the Area of a Circle equal thereto, which Multiply by the leſſer Fruſtums Altitude reſerving the Product.

2. To $\frac{1}{2}$ the Altitude of the greater Fruſtum, add $\frac{1}{6}$ of the Altitude of the leſſer; this done, Multiply the Sum by the former Product, and divide this laſt Product by the Altitude of the greater Fruſtum, the Quotient will be the Content of the leſſer Fruſtum ſought.

Example. In Fig. 10. Let *c a*, the Altitude of the greater Fruſtum, and *d c e* the Diameter of the Fruſtums Baſe be each 24 Inches, *c b* the Altitude of the leſſer Fruſtum 6 Inches, What is the Content of the leſſer Fruſtum (*d c e b*) in Ale-Gallons?

The Area of a Circle whoſe Diameter is equal *d e* (*viz.* 24 Inches) is 1.6042, this Multiplied by *c b*, 6, is 9.6252 which keep: Again, half *a c* is 12, to which add $\frac{1}{6}$ of *c b*, *viz.* 1, the Sum is 13, this Multiplied

Multiplied by 9.6252 is 125.127, and this divided by *a c*, *viz.* 24, the Quotient will be 5.213 the Content of the Fruſtrum in Ale-Gallons. But this may be perform'd with greater expedition by the Inſtrument.

Thus,

1. Set 1 upon D to the Fruſtums Altitude, *viz.* 6, upon C, then againſt the Diameter of the Baſe, *viz.* 24, is 9.625, which keep.

2. Set the Altitude of the greater Fruſtum, *viz.* 24, upon A, to 13 (that is $\frac{1}{2}$ *a c*, more $\frac{1}{2}$ of *c b*) upon B, then againſt 9.6252 upon A is 5.2134, the Content as before.

SECT. VI.

The Uſe of the Inſtrument in the Menſuration of Pyramids and their Fruſtums.

A Pyramid *is a Body contained under ſeveral Planes, ſet upon one Right-Lined Baſe, and meeting in a point at the top which is called the Vertex, alſo in each of theſe Planes a Right Line may be every where applied from the Baſe to the Vertex.*

And under this name *Pyramid* is Comprehended that Pyramidical Body (whoſe Baſe is a Circle) commonly called a Cone.

Problem

Problem I.

To find the Solid Content of a Pyramid.

A *Pyramid* is one third part of a *Prism*, which hath the same Base as the Pyramid, and the Altitude equal thereto:

Therefore,

Multiply the Area of the Base by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude, in either cases the Product will be the Content of the Pyramid.

This is universal for all Pyramids in what ever form their Bases be, and the Content thus found will be Inches, or Gallons, according as the Area of the Base was given in either of these Measures.

1. Example.

In a Square Pyramid (as Fig. 17.) suppose the side of the Base $x\,p$ be 112 Inches, and the Altitude $z\,s$ 90 Inches, what is the Content in Ale-Gallons,

Set 282 } upon } to 112 } upon
then againſt 112 } B { is 44.48 } A,

the Area of the Base in Ale-Gallons. Then, Set 1 upon A, to 30 upon B, and againſt 44.48 upon A is 1334.46 the Content in Ale-Gallons.

2. Ex-

2. Example.

There is a round Pyramid or Cone (as Fig. 18.) ſuppoſe the Diameter at the Baſe *a n o* be 100 Inches, and the Altitude *n x* 180; How many Ale-Gallons may this Cone contain.

Set the Gage-Point for an Ale-Gallon to $\frac{1}{3}$ of the Altitude (*viz.* 60) upon C: then againſt 100 upon D is 1671 the Content required.

Problem II.

To find the Solid Content of the Fruſtum of a Pyramid.

1. BY the Rules aforegoing, find the Area of the greater, and alſo of the leſſer Baſe.

2. Find a Geometrical mean Proportion betwixt theſe two Areas.

3. The ſum of theſe three Numbers being multiplied by one third part of the Altitude, the Product is the Content of the Fruſtum ſought.

Example. Let *a c e o* (in Fig. 18.) repreſent the Fruſtum of a Round Pyramid or Cone, *a o* the Diameter of the greater Baſe is 108 Inches, *c e* the Diameter of the leſſer Baſe 81, and *n v* the Altitude 36 Inches, to find the Content in Ale-Gallons.

1. Set the Gage-Point to 1 in the middle of the Line C; then againſt 108 upon D is 32.48, the Area of the greater Baſe in Ale-Gallons.

2. Set the Gage-Point to 1 at the beginning of the Line C; then againſt 81 upon D, is 18.27, the Area of the leſſer Baſe.

3. Set

3. Set 32.48 upo C, to 32.48 upon D; then againſt 18.27 upon C, is 24.36 the Geometrical mean between the two Area's.

So the Area of the greater Baſe is ——— 32.48
The Area of the leſſer is ——— 18.27
The Geometrical mean betwixt the two Area's is 24.36

The ſum is ——— 75.11

This Multiplied by $\frac{1}{3}$ of the Altitude (*viz.* 12) gives 901.32 the Content of the Fruſtum ſought.

Or by the Pen thus,

1. Square the Diameter of the greater Baſe.
2. Square the Diameter of the leſſer Baſe.
3. Multiply one of theſe Diameters by the other. To this Product add the two Squares, and Multiply the Sum by the Altitude: the laſt Product divided by 1077, Quotes the Content in Ale-Gallons.

SECT.

SECT. VII.

The Uſe of the Rule in Gauging of Brewers Tuns.

THeſe Veſſels are in ſeveral Forms: the moſt uſual may be conſidered under ſome of theſe varieties,

I. *Such whoſe Baſes are equal.*

And if the equal Baſes be alike (that is both Round, both Square, *&c.*) and Parallel, and the ſides of the Tun ſtraight from one Baſe to the other, theſe Tuns are called Priſms. See Problem I. of this Sect.

II. *Such whoſe Baſes are unequal.*

1. If the unequal Baſes be alike (ſuppoſe both Rectangular Parallelograms, both Elliptical, *&c.*) Parallel, Proportional, and alike ſituate, and the ſides of the Tun ſtraight, ſuch Tuns are the Fruſtums of Pyramids. See Problem II. of this Sect.

2. If the unequal Baſes be unlike, ſuppoſe one Round, and the other Elliptical, or if they be alike but not Proportional, or if they be both alike and Proportional, yet if they be not alike ſituate, the Tun is called a Priſmoied, in which 'tis ſtill ſuppoſed the Baſes are Parallel, and ſides of the Tun ſtraight. See Problem VII.

Note,

Note, When the Bases are Proportional it will be,

As the length of the greater Base is to the length of the lesser;

So is the breadth of the greater Base to the breadth of the lesser.

And the Bases are said to be alike situate, when the length of the greater and lesser Bases are both in the same Plain.

Problem I. Figure 11.

There is a Tun whose Bases are both equal and Square, each side of either Base being 138 *Inches, and the depth* 33 *Inches, what is the Content of this Tun in Ale Gallons?* Answer 2228.62.

For,

SET 282 upon B to 138 upon A; then against 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole depth, *viz.* 33 gives 2228.62 the Content required.

Or thus,

Set 16.79 upon D to 33 (the Tuns depth) upon C, then against 138 upon D, is 2228.62 upon C, the Content as before.

Note, 16.79 is the Gage-Point for Square Vessels, it being the Square-Root of 282 the Inches in an Ale-Gallon.

It

It is needleſs to give more Examples of this ſort of Tuns, for let the Baſes be in any form whatſoever, if the Tun have the qualifications above mentioned, the Content is found by Multiplying the Area of the Baſe by the depth; and if the Baſes are Round, as in moſt Tuns they are, the Content may be found at one Operation, as hath been ſhewn in the I. and II. Problem of Sect. IV.

Of Tuns whoſe Baſes are unequal.

Problem II. Figure 16.

There is a Tun, as a, b, c, d, e, f, g, h, *whoſe Baſes are Rectangular Parallelograms, unequal but Parallel, and alike ſituate, and the ſides Proportional, (therefore called the Fruſtum of a Pyramid)* d a *the length of the greater Baſe is* 100 *Inches, and* d c *the breadth* 80; h e *the length of the leſſer Baſe* 85, *and* h g *the breadth* 68, *and the depth of the Tun* 30 *Inches: how many Ale-Gallons may this Tun contain?*

FOR all Tuns of this ſort, this is a general Rule.

Take the Area of the greater Baſe, the Area of the leſſer Baſe, and a Geometrical mean Proportional betwixt the two Area's; the ſum of theſe three multiplied by one third part of the Tuns depth, gives the Content.

Example.

Example either by the Pen, or the Lines A and B upon the Rule.

As 282 is to *d a* 100. *So* is *d e* 80 to the Area of the greater Base, *viz.* —	28.368
As 282 is to *h e* 85. *So* is *h g* 68 to the Area of the lesser Base, *viz.* —	20.496
A Geometrical mean between these two Area's will be found (by Prob. VIII. Sect. 1.) to be —	24.112
The Sum is —	72.976

This Multiplied by one third part of the Tuns depth, *viz.* 10, gives 729.76 the Content in Ale-Gallons.

Problem III. Figure 16.

There is a Tun whose Bases (i l k m *the greater, and* n q o p *the lesser*) *are both Elliptical, Parallel, Proportional and alike situate; the greater and lesser Diameters of each Base being equal to the lengths and breadths of the Bases of the former Tun, and the depth the same; how many Ale-Gallons will this Tun contain?*

THis Elliptical Tun may be conceived to be inscribed in the Tun, whose Content was found in the last Problem; all the given dimensions are the same in both, and the Operation will be the same too,

too, if for the firſt term in the Proportion we take 359.05 inſtead of 282: See the Work.

As 359.05 is to *i k* 100. *So is l m* 80 to the Area of the greater Baſe. —— } 82.281

As 359.05 is to *n o* 85. *So is p q* 68 to the Area of the leſſer Baſe. —— } 16.098

The Geometrical mean between theſe two Area's will be found (by Problem VIII. Sect. I) to be —— } 18.938

The Sum is —— 57.317

This Multiplied by one third part of the depth, *viz.* 10 gives 373.17 the Content of this Elliptical Tun, in Ale-Gallons.

Problem IV.

There is a Tun (as Fig. 17.*) whoſe Baſes are both Square but unequal,* x p *the ſide of the greater Baſe is* 108 *Inches,* r o *the ſide of the leſſer Baſe* 93, *and the depth* s a 30 *Inches, what is the Content in Ale-Gallons?*

THis Tun being the Fruſtum of a Pyramid may be Gaged by the General Rule in Problem II. of this Section.

Or thus;

1. Square the side of the greater Base, and also the side of the lesser Base.

2. Multiply the greater side by the lesser, and to the Product add the two former Squares.

3. The Sum of these three being Multiplied by $\frac{1}{3}$ of the depth, gives the Content in Square Inches, which divided by 282 the Quotient is the Content in Ale-Gallons.

Example.

By the Instrument the Square of *x p* 108 is 11664
The Square of *o r* 93, is —— 8649
108 Multiplied by 93, is —— 10044

Sum is —— 30357
Multiplied by $\frac{1}{3}$ Altitude —— 10

The Product, *viz.* —— 303570

This Divided by 282 quotes 1076.48 the Content sought:

Problem

Problem V. Figure 18.

There is a Conical Tun whose Baſes are both Circular, a o *the Diameter of the greater Baſe is* 108 *Inches,* c e *the Diameter of the leſſer Baſe* 93. *and* u n *the depth* 30 *Inches: what is the Content of this Tun in Ale-Gallons?*

THE Dimenſions being the ſame as in Problem IV. of this Section, proceed as is there directed, the laſt Product will be 303570, this divided by 359.05 quotes 845.48 the Content ſought.

Or thus by the Rule.

Subtract the Diameter of the leſſer Baſe, from the Diameter of the greater, and add half the difference to the leſſer, the Sum is the Diameter in the middle of the Tuns depth; which found,

2. Set the Gage-Point (*a g*) to the Tuns depth upon C.

Then againſt the Diameter upon D, is a fourth number upon C, which keep.

2. Set the Gage-Point to $\frac{1}{3}$ of the Tuns depth upon C.

Then againſt $\frac{1}{2}$ the difference of the Diameters upon D, is a fourth Number, which added to the fourth Number firſt found, is the Content of the Tun ſought.

Example. Figure 18.

a o the Diameter of the greater Base is——	108
c e the Diameter of the lesser Base is——	93
The difference is——	15

The half difference is 7.5 this added to 93 makes 100.5 for the Diameter in the middle of the Tuns depth:

Therefore,

1. Set the Gage-Point to 30 the Tuns depth, then against 100.5 the Diameter is 843.91 which keep.

2. Set the Gage-Point to $\frac{1}{3}$ of the Tuns depth, that is 10; then against 7.5 the semidifference of the Diameters is 1.56 this added to 843.91 (the Number first found) the sum is 845.47 the Content sought, which agrees very well with the former Rule.

Problem VI.

To find the Content of a Pyramidal Tun whose Bases are in the form of any of the first 12 *Regular Polygons, one side of each Base, and the Tuns depth being given.*

THE Area of each Base of any such Tun may be found by Problem VIII. Section III.

A Geometrical mean betwixt these Area's may be found by Problem VIII. Section I.

The sum of the two Area's and the Geometrical mean, being Multiplied by $\frac{1}{3}$ of the Tuns depth gives the Content.

Or

Or thus,

By the Rule in the laſt Problem, Square the ſide of each Baſe, Multiply the greater ſide by the leſſer; then Multiply the ſum of theſe three by $\frac{1}{3}$ of the Tuns depth, and Divide this laſt Product by a Number in the following Table, proper to the Polygon given, the Quotient will be the Content in Ale-Gallons.

Example. Let there be a Pyramidal Tun, whoſe Baſes are both in the form of a Pentagon (or figure of five equal ſides) and let the ſide of the greater Baſe be 108. the ſide of the leſſer 93, and the depth 30 Inches; all the ſame as in the laſt Problem: therefore according to the Rule and Example there laid down, the laſt Product will be 303570, this Divided by 163.91 (the proper Diviſor for a Pentagon) the Quotient will be 1852.05 the Content in Ale-Gallons: and ſo for any other, taking the proper Diviſor out of this Table.

Names of Polygons.	*Diviſors for Ale-Gallons.*	*Number of Sides.*
Trigon	651.25	III
Tetragon	282.00	IV
Pentagon	163.91	V
Hexagon	108.55	VI
Heptagon	77.59	VII
Octogon	58.41	VIII
Nonogon	45.62	IX
Decagon	36.65	X
Undecagon	30.11	XI
Dodecagon	25.18	XII
Cone	359.05	

To find the Content of any Tun whose Parallel Bases are Rectangular Parallelograms, or Ellipsis, the sides of the Tun being straight from the top to the bottom, whether the Bases be alike or unlike, Proportional or disproportional, alike situate or Inverted, by one General Rule.

Problem VII.

Let Fig. 16. represent a Tun called a Prismoid, the Bases are Rectangular Parallelograms unequal and disproportional, but parallel and alike situate; a d *the length below* = 150 *Inches, and* a b *the breadth* 100, e h *the length above* 90, *and* e f *the breadth* 70, *and the depth* 30: *What is the Content in Ale-Gallons?*

The Rule.

1. TO *the greater length* a d *add* $\frac{1}{2}$ *the lesser length* e h, *and Multiply the sum by the greater breadth* a b, *reserving that Product.*

2. *To the lesser length* e h, *add* $\frac{1}{2}$ *the greater length* a d, *and Multiply the sum by the lesser breadth* e f; *add this Product to the former reserved Product.*

3. *Multiply the sum of these two Products by* $\frac{1}{3}$ *of the Tuns depth, and divide the last Product by* 282, *the Quotient will be the Content of the Tun in Ale-Gallons.*

Example.

Example.

1. *a d* 150 added to $\frac{1}{2}$ *e b* 45, the ſum is 195, this Multiplied by *a b* 100, the Product is 19500 which keep.

2. *e b* 90 added to $\frac{1}{2}$ *a d* 75 the ſum is 165, this Multiplied by *e f* 70, the Product is 11550; this added to the former Product, *viz.* 19500 the ſum is 31050, this Multiplied by $\frac{1}{3}$ of the depth, *viz.* 10, the Product is 310500, and this Divided by 282 gives 1101.06 the Content in Ale-Gallons.

The truth of this Rule will plainly appear if we duly conſider Fig. 13. in which *a b c d* is equal to the greater Baſe, and *e f g h* equal to the leſſer Baſe of the Tun in the laſt Problem, (which was repreſented by Fig. 16.)

In this Fig. 13. *b i e v* is made equal to *f e h g*.

Alſo, *a b* 100 leſs by *e f* 70, is equal to *a i* 30.

And *a d* 150 leſs *e h* 90, is equal to *i d* 60, the Altitude is 30 as before.

The Lines being drawn, the whole Solid is Compoſed of theſe parts, *viz.* three Priſms and a Pyramid.

The $\left\{\begin{matrix}1\\2\\3\end{matrix}\right.$ Priſm is $\left\{\begin{matrix}b\,i\,e\,u\,f\,e\,h\,g\\i\,a\,s\,o\,h\,e\\o\,v\,c\,n\,b\,g\end{matrix}\right.$

The Pyramid is *o s n d h*.

The Content of theſe four Solids being added together, gives the Content of the whole Solid *a b c d e f g h*; for the whole is equal to all its parts taken together. (Axiom 19. 1. *Euclid.*)

And the Content of theſe may be found thus:

1. Multiply *f g* 90 by *f e* 70, the Product is 6300; this Multiplied by *b f* the Altitude, *viz.* 30, gives 189000 the Content of the Prism, *b i o v f e h g.*

2. Multiply *a i* 30 by *a s* 90, the Product is 2700, this Multiplied by $\frac{1}{2}$ the Altitude, *viz.* 15, is 40500 the Content of the Prism *i a s o h e.*

3. Multiply *e n* 70 by *n o* 60, the Product is 4200; this Multiplied by 15 is 63000, the Content of the Prism *o v c n h g.*

4. Multiply *s d* 60 by *s o* 30, the Product is 1800, this Multiplied by $\frac{1}{3}$ of the Altitude, *viz.* 10, gives 18000 the Content of the Pyramid *s o n d h.*

So the Content of the 1	Prism is	189000
2		40500
3		63000
Pyramid is		18000
The Sum		310500

Is the Content in Square Inches, and is exactly the same with the Content found by the Rule given in the last Problem.

The truth of this Rule being thus demonstrated, we shall shew its further usefulness in the following Problems.

Problem

Problem VIII. Figure 16.

There is a Tun (called a Cylindroid) whose Baſes are both Elliptical, unequal, and diſproportional (that is, there is not the ſame proportion betwixt the Axis and Diameter of the greater Baſe, as is betwixt the Axis and Diameter of the leſſer) let the given Dimenſions be the ſame as in the laſt Problem, viz.

$n\,o = 90 : q\,p = 70$ } Rectangular } above
$i\,k = 150 : l\,m = 100$ } Conjugates } below.

And the depth 30 *Inches: what is the Content of this Tun in Ale-Gallons?*

For the Content of this Tun Work by the General Rule, and according to the Example given in the laſt Problem, only inſtead of 282 Divide the laſt Product by 359.05: So in this Example (the Numbers being the ſame as in the laſt) the laſt Product will be 310500 this Divide by 359.05 quotes 864.78 the Content in Ale-Gallons.

Problem IX. Figure 15.

There is a Tun whose Bases are Rectangular Parallelograms and Parallel, but neither equal nor alike situate (for g h *the breadth above is posited directly opposite to* a d, *the length below) how many Ale-Gallons will this Tun contain?*

The Dimensions are

h g Breadth = 68 : *h e* Length = 85 above
d a Length = 100 : *d c* Breadth = 80 below.
The Depth 30 *Inches.*

If the Bases of this Tun were alike situate (as is supposed in Problem II. of this Section) the Content would be 729.76 Ale-Gallons as was there shewn; but the Bases being Inverted as aboveſaid, the Content will be found to be 735.81 Ale-Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus *h g* 68 added to $\frac{1}{2}$ *d a* 50 is 118, this Multiplied by *h e* 85 the Product is 10030.

Again, *d a* 100 added to $\frac{1}{2}$ *h g* 34 is 134, this Multiplied by *d c* 80, the Product is 10720, the sum of these two Products is 20750, which Multiplied by $\frac{1}{7}$ of the depth, *viz.* 10, gives 207500 the Content in Square Inches, this Divided by 282, The Quotient is 735.81 the Content in Ale-Gallons.

Or if the last Product be Divided by 359.05. the Quotient will be 577,91 the Content of an Elliptical Tun, whose Axis above is directly opposite to the

the ſhorter Diameter below; and ſuch an Elliptical Tun may be conceived to be inſcribed in the other, the reſpective Axi's and Diameters, Lengths and Breadths, and alſo the Depth being the ſame in both.

Problem X. Figure 14.

To find the Content of a Tun whoſe Baſes are unlike, ſuppoſe one Square, and the other Oblong; or one Round, and the other Elliptical, but Parallel one to the other, and the ſides of the Tun ſtraight.

The Dimenſions are,

Breadth.	Length.	
$e f = 74$:	$f g = 100$	Rectangular Conjugates above
$d c = 100$:	$c b = 100$	Rectangular Conjugates below.

Depth 30 *Inches.*

By the General Rule in Problem VIII. Work thus:

1, To $e f$ 74 add $\frac{1}{2}$ $d c$ 50 the ſum is 124, this Multiplied by $f g$ 100 is 12400.

2. To $d c$ 100 add $\frac{1}{2}$ $e f$ 37 the ſum is 137, this Multiplied by $c d$ 100 is 13700, this added to the former Product, *viz.* 12400, makes 26100, and this Multiplied by $\frac{1}{7}$ of the Depth, *viz.* 10, the Product is 261000, this Divided by 282 gives 929.53 the Content of this Tun in Ale-Gallons, ſuppoſing the Baſes were one Square, and the other Oblong, as the Figure repreſents them:

But if the Baſes were one Round, and the other Elliptical, the laſt Product, *viz.* 261000 muſt be Divided

Divided by 359.05 and in this Case the Content will be 726.91.

By what hath been said I presume it will not be difficult to find the whole Content of any Tun, whose Bases are Parallel, and Sides straight from top to bottom.

Sect. VIII.

Shewing how to find the Content of any Tun from Inch to Inch, or what any Tun will contain upon every Inch of its depth, which is commonly called Inching of a Tun.

Problem I.

Suppose a Tun whose Bases are both Round, or both Square and Parallel one to another, and in the sides a straight Line may be every where applied from Base to Base: let the Diameter or side of the greater Base be 187.2 *Inches, the Diameter or side of the lesser Base* 180 *and the depth* 12 *Inches, to Inch this Tun (be it Round or Square) from the lesser Base downward.*

Rule,

From the Side or Diameter of the greater Subtract the Side or Diameter of the lesser Base, Divide their difference by the Tuns depth, and call the Quotient *a*.

For the Side or Diameter of the lesser Base put *b*.

Mul-

Multiply the Number *b* by the Number *a*, and call the Product *c*. Then,

1. To the Square of *b* add the Number *c*, and one third part of the Square of *a*, the sum of these three will shew the Content of the first Inch of the Tun.

2. Multiply one third part of the Square of *a* by 2, to this Product add the Number *c*, then Multiply the sum by 2, and to this Product add the Square of *b*, Multiply this sum by 2, then will this last Product shew the Content of the two first Inches of the Tun.

3. Multiply one third part of the Square of *a* by 3, then Add and Multiply as in the last, the third and last Product will shew the Content of the three first Inches of the Tun.

4. Having got these three Numbers, Subtract the first from the second, and likewise the second from the third, then the first Number, and each of these differences or remainders being severally Divided by 282 for Square Tuns, or by 359.05 for Round Tuns, the respective Quotients will be the Content (in Ale-Gallons) of the first, second and third Inches of the Tun, by which a Table may be made for the whole Tun as in the Example following.

Example of a Square Tun.

The side of the greater Base	187.2
The side of the lesser Base	180.0
The remainder is	7.2

This Divided by the Tuns depth, *viz.* 12 quotes .6 for the Number *a*.

The

The Number *b* (which is the Side of the leſſer Baſe) is 180, this Multiplied by the Number *a viz.* .6 gives 108 for the Number *c*.

1. The Square of *b* is	32400
Number *c* is	108
$\frac{2}{3}$ Of the Square of *a* is	.12
The ſum is	32508.12

For the firſt Inch of the Tun.

2. $\frac{1}{3}$ Of the Square of *a* is	.12
Multiply by	2
	.24
Number *c* added	108.00
Sum is	108.24
Multiply by	2
	216.48
Square of Number *b* added	32400.00
Sum is	32616.48
Multiply by	2
Sum is	65232.96

For the two firſt Inches.

3.

3. $\frac{1}{3}$ Of the Square of a	.12
Multiply by	3
	.36
Number c add	108.00
Sum is	108.36
Multiply by	3
	325.08
Square of b added	32400.00
Sum is	32725.08
Multiply by	3
Sum is	98175.24

For the three first Inches.

Two first Inches	65232.96
First Inch	32508.12
Rest for second Inch	32724.84

Three first Inches	98175.24
Two first Inches	65232.96
Rest for third Inch	32942.28

Now

Now these three Numbers.

viz.	32508.12	Divided by 282 quotes	115.277
	32724.84		116.045
	32942.28		116.816

For the Content (in Ale-Gallons) upon the first, second, and third Inches of this Tun, supposing the Bases Square.

And if the said Numbers be Divided by 359.05 the respective Quotients will

Be	90.539	the Content of the first, second and third Inches of the Tun, when its Bases are Round.
	91.142	
	91.747	

Now by these Numbers the following Table is composed for the Round Tun. Thus,

Take the first Inch from the second, the remainder or difference is .603, which set down in the third Column, against the space betwixt the first and second Inches. In like manner take the second Inch from the third, the remainder is .605, which place in the third Column under the other, then take .603 from .605, there remains .002 for the second Difference, which is always the same as in the fourth Column, this second Difference added to the second Number in the third Column, *viz.* .605, makes .607, and this added to the Content of the third Inch, *viz.* 91.747 makes 92.354 the Content of the fourth Inch, and so by a continual addition of the second Difference to the first, and of their sum to the Inch last found, the Table is made.

Depth

Depth in Inches.	*Content of every Inch in Ale-Gall.*	*First Differ.*	*Second Differ.*
1	90.539	.603	
2	91.142	.605	.002
3	91.747	.607	.002
4	92.354	.609	.002
5	92.963	.611	.002
6	93.574	.613	.002
7	94.187	.615	.002
8	94.802	.617	.002
9	95.419	.619	.002
10	96.038	.621	.002
11	96.659	.623	
12	97.282		

Sum---1126.766---Content of the Tun.

In this Example we have made a Table for the Round Tun, and the Operation would have been the ſame for the Square Tun, had we taken the three firſt Inches of *that* inſtead of *theſe*.

Note, When we Inch a Tun by this Rule, we ſuppoſe it to be Regular, having all the qualifications expreſt in this Problem. But for as much as moſt Tuns have ſome irregularity or unevenneſs in their Sides, I ſhall here lay down a more Practical Rule for Inching of Tuns.

Problem

Problem II.

There is a Tun whose Diameter at the bottom is 108 *Inches, the Diameter at the top* 93, *and the depth* 30, *how many Ale-Gallons will this Tun contain upon every Inch of the depth?*

TO resolve this Question, there must first be known the whole Content of the Tun, and also the Area of the mean Diameter in the middle of every 10 or 12 Inches of the depth, and these may be thus found.

Take (with the long Rule and Cane, or some other convenient Instrument) a Diameter in the middle of every 10 Inches of the Tuns depth, and suppose in this Example the first (at 5 Inches from the top) be 95.5, the second (at 15 Inches from the top) 100.5, and the third 105.5, these I set down as in the following Table.

This done, we may consider the Tun either as Round or Square, and first to Inch it as a Round Tun.

Set the Gage Point (*a g*) to 1; then against the several Diameters already found, you have their respective Area's; So the first Area will be 25.40, the second 28.13, the third 31.00, these I set down in the Table against their Diameters, and having added them together, the Sum is 84.53, this Multiplied by 10 (or which is all one, remove the Prick one place towards the Right hand) the Product is 845.3, the whole Content of the Tun.

Note,

Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no use of Multiplication or Division in Inching the Tun, as there will be when the Diameters are taken in the middle of every Foot or half Foot, for here the Area's of the several Diameters do not only shew the mean Inch in the middle of every 10 Inches, but also the whole Content of those 10 Inches, if you remove the Prick but one place more towards the Right hand. Thus the mean Inch of the last 10 Inches is 31.00, but if the Prick be removed as aboveſaid, it is 310, which is the Content of the Tun at 10 Inches deep, I therefore reduce the ſeveral Contents in the third Column into Barrels, Firkins and Gallons of Beer meaſure, and ſet them down in the fourth Column, theſe added together are 23 Barrels, 1 Firkin, 8 Gallons and 3 tenths of Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tuns depth.	*Mean Diameters in the middle of every 10 Inch of the Tuns dep.*	*Mean Area's or Contents of every 10 Inch of the Tuns dep.*	*Content of every 10 Inches Beer Meaſure.* B.	F.	G.
10	95.5	25.40	2	0	2.0
10	100.5	28.13	7	3	2.3
10	105.5	31.00	8	2	4.0
30		84.53	23	1	8.3

By theſe Numbers thus found it will be eaſie to make a Table which ſhall ſhew the quantity of Liquor contain'd in the Tun at any depth, for which obſerve the following directions.

1. Set

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons, and Parts, in their reſpective Columns, and at the top of the firſt Column againſt the Content ſet 0, and under it the Tuns depth in Inches 1, 2, 3, 4, 5, *&c.* this done take the firſt mean Inch, *viz.* 25.4 and reduce it into Barrels, Firkins, and Gallons, Beer Meaſure, it makes 0 : 2 : 7 : 4 : ſet this upon a ſcroll of Paper, and Subtract it continually from the whole Content till you come at the 10th. Inch of the Tuns depth, which done, if you miſtake not, the remainder there will be 16 : 1 : 6 : 3.

2. Take the ſecond mean Inch, *viz.* 28, 13, this being reduced as the former is 0 : 3 : 1.13, Subtract this continually from the laſt remainder, 'till you come at 20 Inches deep, and there your remainder will be 8, 2, 4.

3. Take the third and laſt mean Inch, and reduce and Subtract it as before, and if your Work be right there will be no remainder at the laſt Inch or bottom of the Tun; now the Table being made you may caſt away the Fractions as uſeleſs. The uſe of this Table is obvious to the meaneſt Capacity, without any further explanation: For if you come to this Tun and

In.	B.	F.	G.
0	23	1	8.30
1	22	3	0.9
2	22	0	2.5
3	21	1	4.1
4	20	2	5.7
5	19	3	7.3
6	19	0	8.9
7	18	2	1.5
8	17	3	3.1
9	17	0	4.7
10	16	1	6.3
11	15	2	5.17
12	14	3	4.04
13	14	0	2.91
14	13	1	1.78
15	12	2	0.65
16	11	2	8.52
17	10	3	7.39
18	10	0	6.26
19	9	1	5.13
20	8	2	4.00
21	7	3	0
22	6	3	5
23	6	0	1
24	5	0	6
25	4	1	2
26	3	1	7
27	2	2	3
28	1	2	8
29	0	3	4
30	0	0	0

find

find 2 Inches of the depth dry, the quantity of Liquor then in the Tun is 22 B. 0 F. 2 G. If 7 Inches be dry, there remain in the Tun but 18, 2, 1, *&c.* Thus for Round Tuns.

Now ſuppoſe the Tun laſt mentioned were Square, and the ſides at bottom and top equal to the Diameters of the laſt, and the depth the ſame, the Diameters or ſides in the middle of every 10 Inches, of the Tuns depth will be the ſame as in the laſt, by which the ſeveral mean Inches, and alſo the whole Contents of every 10 Inches are found to be as in this Table.

Parts Tuns depth.	*Sides of the Tun in the middle of every 10 Inches.*	*Mean Area's or Content of every 10 Inches.*	*Content of every 10 Inches Beer Meaſure.* B. F. G.
10	95.5	32.34	8 2 8.4
10	100.5	35.81	9 2 6.1
10	105.5	39.46	10 3 7.6
30		107.61	29 2 4.1

For, Set 282 upon B, to 95.5 the firſt Diameter upon A, then againſt 95. 5 upon B is 32.34 the firſt mean Inch, and after the ſame manner find the ſecond and third, which being reduced as in the laſt Example, I find the whole Content to be 1076.1 Ale-Gallons (or which is the ſame thing) 29 B. 2 F. 4.1 G. this being done you may proceed to Inch this Tun by the ſame directions which were given in the laſt Example.

Problem

Problem III.

To Inch a Tun whose Bases are unlike, viz. *the lower Circular, and the upper Elliptical, suppose the Diameter of the Circular Base be* 100 *Inches, the Axis (or longest Diameter) of the Elliptical Base* 100, *and the shortest* 90, *and the Tuns depth* 20 *Inches.*

TO Inch this Tun, I take (with the long Rule) the cross Diameters in the middle of every 4 Inches of the Tuns depth: Thus at 2 Inches from the top I find the longest Diameter 100, and the shortest 91, in the middle of the next four Inches (which is 6 Inches from the top) the longest Diameter is 100, and the shortest 93, and thus I find the rest which are as in this Table is express'd: Now having these Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tuns depth may be thus found.

Set 359.05 upon B, to the longest Diameter, *viz.* 100 upon A: then against the shorter Diameter upon B, is the Area in Ale-Gallons; thus against 91 is 25.34, the Area in the first 4 Inches: against 93 is 25.88 the second Area. In like manner against 95, you have 26.47 the third mean Inch: And so of the others as in the Table.

Depth.

Depth.	Axis.	Shorter Diam	Mean Inch in Ale-Gallons.	Content of every 4 Inches of the Tuns Depth.
4	100	91	25.34	2 3 2.36
4	100	93	25.88	2 3 4.52
4	100	95	26.47	2 3 6.88
4	100	97	27.02	3 0 0.08
4	100	99	27.57	3 0 2.28
29			132.28	14 . 2 . 7.12

Now if the mean Inches be severally Multiplied by 4, and these Products reduced into Barrels, Firkins, and Gallons, Beer measure, they will give the Numbers in the last Column, which added together are 14 Barrels, 2 Firkins, 7 Gallons, and .12 parts of a Gallon.

Set this down for the first Number of your Table, and then reduce the first mean Inch into Barrels, Firkins, and Gallons, and Subtract it 4 times from the whole Content and remainders; do the like with the other mean Inches, and so complete the Table as in the last Example.

Problem IV.

To find the Drip or Fall of a Tun, and to make allowance for the same in Inching the Tun.

As for Instance.

ADmit the Tun last mention'd were so plac'd, that when the bottom is but just cover'd on one

one ſide, the Liquor is 4 Inches deep on the ſide oppoſite; How much muſt be allow'd for the fall of this Tun.

By the operation aforegoing the Area of the mean Diameter in the middle of the 4 laſt Inches (that is at 2 Inches from the bottom) was found to be 27.57, this Multiplied by 4, is 110.28 Ale-Gallons, the half of this is 55.14, and ſo much may be allow'd for the fall of the Tun, I therefore reduce this 55.14 into Barrels, Firkins, and Gallons Beer Meaſure, it gives 1 : 2 : 1.14: this taken from 14 : 2 : 7.12 the Content of the Tun when it ſtands level, there remains 13 Barrels, 0 Firkins, 5 Gallons, and .98 parts: and ſo much will the Tun contain, when the fall is four Inches, I therefore ſet this at the top of the Table, then reducing the firſt mean Inch into Barrels, Firkins and Gallons, I Subtract it four times from the whole Content, the remainder is 10 : 1 : 3 62: Do the like with ſecond, third and fourth mean Inches, and you find the remainder at 16 Inches dry to be 1 : 2 : 1.14: and ſo much it takes to cover the bottom as was before obſerved.

0	13	0	5.98
1	12	1	7.64
2	11	3	0.30
3	11	0	1.96
4	10	1	3.62
5	9	2	4.74
6	8	3	5.86
7	8	0	6.98
8	7	1	6.10
9	6	2	8.63
10	6	0	0.16
11	5	1	0.69
12	4	2	1.22
13	3	3	1.20
14	3	0	1.18
15	2	1	1.16
16	1	2	1.14

As for ſuch Tuns as are ſuppoſed to be the Fruſtums of Spheroids, Parabolick Conoids, Hyparabolick Conoids, or Parabolick Spindles, I ſhall give Rules for Gauging and Inching of theſe in the Appendix, where I ſhall alſo endeavour to explain the nature of thoſe Figures and whence they are derived.

Problem

Problem V. Figure 20.

To Gauge a Copper and make allowance for the Crown; let a, b, c, d, e, a, *repreſent a Copper whoſe Content is required.*

1. TO find the ſeveral Diameters and Depth of the Copper, together with the Altitude of the Crown.

Take a ſmall Cord or Thread, make one end faſt at *a*, and extend the other to the oppoſite ſide of the Copper at *e*, where make it faſt, or cauſe a perſon to hold it very ſtraight; this done, ſet one end of the Inſtrument in the bottom of the Copper at *b*, and move it to and fro till you find the neareſt diſtance to the Thread as at *o*, this diſtance *b o*, is the depth of the Copper, ſuppoſe it be 44 Inches.

In like manner ſet the end of the Rule upon the top of the Crown at *c*, and take the neareſt diſtance to the Thread as *c v*, ſuppoſe it be 36 Inches, this Subtracted from *b o* 44, the remainder 8 is the Altitude of the Crown.

To find *b d* the Diameter of the bottom of the Crown.

Meaſure *a e*, the Diameter at the top, admit it be 115 Inches, then hold a Thread ſo as a Plummet at the end thereof may hang juſt over *b*; by this means you may find the diſtance *a o*, and on the other ſide *e i* ſuppoſe each be 14.5, add theſe together and Subtract their Sum (*viz.* 29) from *a e*, 115 the remainder 86 is equal to *b d*, the Diameter at the bottom of the Crown: the Diameter which touches the top of the Crown may be found by the Inſtrument, ſuppoſe it be 91.4 Inches.

Now to find the Content of the Copper from the Crown upwards, (that is, the part *abcie*) the depth *vc* being 36 Inches, you may take a Diameter in the middle of every 6 Inches betwixt *v* and *c*, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Area's in Ale-Gallons, found by the Gage-Point as before; the fourth Column ſhews the Content of every 6 Inches in Barrels, Firkins and Gallons, Beer meaſure: Theſe added together are 29 Barrels, 3 Firkin, 2.3 Gallons.

And ſo much will the Copper contain after the Crown is cover'd.

Now if the Crown be taken for the Fruſtum of a Sphere, the Content (by the third Problem of Section V.) will be found to be 83.3 Gallons.

But the Content may be more readily found and very near the truth: thus,

The Diameter *bd*, was found to be 86, the Area to this Diameter is 20.6, this Multiplied by $\frac{1}{2}$ the Crowns Altitude, *viz.* 4, gives 82.4, the Content of the Crown, this Subtracted from the Content of the part or portion *hcidnb*, the remainder is the quantity of Liquor that it will require to cover the Crown; the Content of the part *hcidnb* may be thus found, *hi* is 91.4, and *bd* is 86, theſe added together are 177.4, the half 88.7 may be taken for a mean Diameter, the Area to this Diameter is 21.9, this Multiplied by 8, the Altitude, gives 175.2 the Content.

Now Subtract 82.4 (the Content of the Crown) from 175.2, the remainder is 92.8, or 2 : 2 : 2.8, this added to the Content firſt found, *viz.* 29 : 3 : 2.3, gives 32 : 1 : 5 the Content of the whole Copper.

Parts

Parts of the depth.	*Diameter.*	*Mean Inches.*	*Content of every 6 Inches in* B. F. G.
6	113.1	35.62	5 3 6.7
6	109.2	33.21	5 2 1.2
6	105.28	30.86	5 0 5.1
6	101.36	28.61	4 3 0.6
6	97.33	26.37	4 1 5.2
6	93.4	24.29	4 0 1.7
36	Sum		29 3 2.5
To cover the Crown			2 2 2.8
Content Copp.			32 1 5.3

The Content being thus found you may proceed to Inch the Copper by the same directions which were given for Inching of Tuns in the last Problem.

Problem VI.

The Diameter of any Round or Side of any Square Mash-Tun, being given, to find how many Gallons of Mault it will contain at one Inch deep.

THE Corn-Gallon (which is the measure we here account by) is supposed to contain 272 25 Cubick Inches.

But Mr. *Dary* in his Complete Gauger saith, that an indifferent sort of Mault, and of an indifferent Grinding, when three Worts shall have passed through it, is more compact by about a sixth part; he there-

fore concludes that 227 may be a proper Diviſor for Square Maſh-Tuns, and 388 for Round ones.

1. *For Round Maſh-Tuns.*

The Diameter of that Circle whoſe Area is 227 is 17, and this may be called the Gauge-Point for Round Maſh-Tuns, and to find the Content of any ſuch:

Set 17 upon D, to 1 upon C; then againſt any Diameter upon D, you have the Content in Gallons at one Inch deep: thus if the Diameter were 113, the Area or Content at one Inch deep would be 44.3 that is, 5 Buſhels and 4 Gallons.

2. *For Square Maſh-Tuns.*

If a Maſh-Tun be Square or Oblong, the Content at one Inch deep may be thus found: Say,

As 227 is to the Length,
So is the Breadth to the Content.

Example.

Suppoſe the length 112 Inches, and the breadth 80, how many Gallons will this Tun contain at one Inch deep? Anſwer 39.4: For,

Set 227 upon A, to 80 upon B; then againſt 112 upon A, is 39,4 the Content upon B.

SECT.

SECT. IV.

Of Cask Gauging.

BEfore we can find the Content of a Cask, we muſt know the Dimenſions of it, *viz.*

The { Bung Diameter, / Head Diameter, / Length, } in Inches.

And when theſe are known, the form or ſhape of the Cask muſt be conſidered, for although the Diameters (abovesaid) and the length of one Cask may be equal to thoſe of another, yet one of theſe Casks may contain ſeveral Gallons more than the other, and therefore the Content of all Casks cannot be found by one and the ſame Rule.

Now moſt Writers in Treating on this Subject, have taken it for granted, that every common Cask (as Butts, Pipes, Hogſheads, Barrels and ſuch like) is in the form of one of the Solids, *viz.*

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a Parabolical Spindle.
3. The middle Fruſtum of two Parabolical Conoids, abutting upon one common Baſe, or
4. The middle Fruſtum of two Cones abutting upon one common Baſe.

They have not (as I remember) told us what they mean by all thoſe terms, but have laid down Rules for finding the Content of theſe Solids, and by theſe

Rules they ſuppoſe the Content of any Cask may be alſo found.

As for inſtance,

If the Dimenſions of a Cask be

Bung Head } Diameter — { 34 / 24 } Inches:
Length ——— 48

By the Rules given by Mr. *Dary*, and others, the Content of this Cask if taken for

			Ale-Gall.
The middle Fruſtum of	1 a Spheroid	is	128.71
	1 a Parabolical Spindle		127.64
	2 Parabolic. Conoids		115.77
	2 Cones		113.53

Dr. *Newton* in his Coſmography page 81. gives a Rule by which the Content of this Fruſtum of a Parabolical Spindle will be but 118.37, and according to others it ſhould be but 110.87 Gallons.

But they all agree in giving true Rules for the firſt and laſt of theſe Solids, *viz.*

The middle Fruſtum of { a Spheroid, two Cones.

And theſe may both be repreſented by Figure 21.

A Cask which is the middle Fruſtum of a Spheroid is ſhewn by the outermoſt Line *d*,*o*,*a*,*o*,*d*,*f*,*c*,*o*,*b*,*o*,*c*,*e*,*d*, the two ſtraight Lines *d*, *e*, *c*, and *d*, *f*, *c*, are the Heads of the Cask, and the Curved Lines *d*,*o*,*a*,*o*,*d*, and *c*, *o*, *o*, *c*, the Staves.

If

If a Cask (can) be in the form of the middle Fruſtum of two Cones, it may be repreſented by the ſtraight Lines *d, u, a, u, d*, and *c, u, b, u, c*; the Heads *d, e, c*, and *d, f, c*, the Bung Diameter *a, b*, and the Length *e f*, are all the ſame in both theſe Casks, and yet 'tis evident by the Figure it ſelf, that one will contain more than the other, and by the Example above Cited the Difference is 15.16 Gallons, for the Spheroid Contains 128.71 Gallons, and the Fruſtum of two Cones but 113.55.

There are very few (if any) Casks whoſe Diameters and Length are equal to thoſe above named, that will contain more than 128.71 Gallons (the Content of the Spheroid) nor leſs than 113.55 (the Content of the Fruſtum of two Cones) ſo that all the varieties that can happen will fall betwixt theſe two, and the Staves of any Cask differing from theſe will lie between the Lines *d, o, a*, and *d, u, a*, in Figure 21.

Now for the other two Solids, *viz.* the middle Fruſtum of a Parabolical Spindle, and the middle Fruſtum of two Parabolical Conoids; according to the Authors above cited, the Content of the firſt is 127.64 Gallons, and the ſecond 115.77.

Whether the Rules they have given for finding theſe Contents be true or falſe, I ſhall examine in the Appendix.

But ſuppoſing they are true, yet they do not anſwer the various caſes that may happen in common Practice, for in the Example above, the Content of a Cask taken, as the middle Fruſtum of a Parabolical Spindle is but 1.07 leſs than the Content of the Fruſtum of a Spheroid, and the Content of the Fruſtum of two Parabolical Conoids is but 2.22 more than the Fruſtum of two Cones.

Now I presume that no Man will pretend to distinguish which Notion a Cask must be taken in, whether as the Frustum of a Speroid, or as the Frustum of a Parabolical Spindle, when the Contents differ but one Gallon in 128.

Moreover, here is almost 12 Gallons difference betwixt the Content of the Frustum of a Parabolical Spindle, and the Frustum of two Parabolical Gonoids: So that I may modestly affirm that there are many Casks whose Content cannot be found by any of these Rules; for it must be granted than a Cask whose Diameters and Length are equal to those in this Example, may contain some certain quantity betwixt 127 Gallons, and 115.

As suppose 123, I say if this Cask were to be Gauged by any of the Rules above cited, they will give the Content at least 4 Gallons too much, or 7 Gallons too little.

This premised, I come now to lay down some short Rules by which the Content of any Cask may be found very near the truth.

Problem

Problem I. Figure 21.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, intercepted between two plains Parallel, cutting the Axis at Right Angles, let a b *the Diameter at the Bung be* 34 *Inches,* d c *the Diameter at the Head* 24, *and the Length* e f 48 *Inches, How many Ale or Wine-Gallons will this Cask contain?*

Rule.

TO *the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the difference of the ſaid Squares, the Sum of theſe Multiplied by the Length, and this Product divided*

By { 1077 / 882 } Quotes the Content in { Ale / Wine } Gallons.

Example.

Square of *a b* 34 is	1156
Square of *d c* 24 is	576
Sum is	1732
Half Sum	866
Half the difference of the Squares is	290
Sum is	2888

Which Multiplied by 48 (the Length of the Cask) gives 138624, this Divided by 1077 Quotes 128.71 the Content in Ale-Gallons.

Or Divided by 882 the Quotient will be 157.17 the Content in Wine-Gallons.

Note, the Common Rule for Gauging the Fruſtum of a Spheroid, (by taking twice the Square of the Bung, and the Square of the Head, *&c.*) is ſomewhat a ſhorter way than this: but this Rule being more agreeable to the Work in the following Problem, I have thought fit to inſert it here: the Reader may uſe which he pleaſe.

Problem II.

Suppoſe a Cask (of the ſame Diameters, and Length with the former) ſhall have its Staves betwixt the Bung and Head leſs Curved, ſo as to leave out about a third part of the ſpace betwixt the Lines d o a, *(which repreſents the Spheroid) and* d u a, *as in Figure* 22. *How many Gallons will this Cask Contain?*

Rule.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add three tenth parts of the Difference of the ſaid Squares, the Sum of theſe Multiplied by the Length, and the laſt Product Divided by 1077, *or* 882, *Quotes the Content in Ale or Wine-Gallons reſpectively.*

Example

Example. *The Dimensions as above.*

Sum of the Square is	1732
Half Sum is	866
The Difference of the Squares Multiplied by .3 is	174
Sum of these is	2772

This Multiplied by 48, gives 133056, and this Divided by 1077 quotes .123.54 the Content in Ale-Gallons, or by 882, the Quotient will be 150.85 the Content in Wine-Gallons.

Problem III. Figure 19.

There is a Cask whose Staves, between the Bung and Head, have but little Curveture, lying nearer to the straight Line a u d, *than to the Curved Line* a o d, *what is the Content of this Cask in Ale or Wine-Gallons?*

Rule.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add one tenth of the Difference of the said Squares, the Sum of these Multiplied by the Length, and this Product Divided as above directed, quotes the Content in Ale or Wine-Gallons respectively.

Example.

Example. (*Dimensions as above.*)

Sum of the Square is	1732
Half Sum is	866
One tenth of the Difference is	58
Sum of these is	2656

Which Multiplied by 48, gives 127488, this Divided by 1077 the Quotient is 118.37, the Content in Ale-Gallons; or by 882 quotes 144.54, the Content in Wine-Gallons.

Problem IV. Figure 21.

To find the Content of a Cask in the form of the middle Frustum of two Cones abutting upon one common Base (if any such can be) it may be represented by the straight Lines d u a, a u d.

Rule.

From the Sum and half Sum of the Squares of the Bung and Head Diameters, Subtract half the Square of the Difference of the Diameters themselves, Multiply the remainder by the Length, and Divide this Product as in the last, the Quotients will be the Content in Ale or Wine-Gallons.

Example.

Example. (*Dimensions as above,*)

Sum of the Square is ——— 1732
Half Sum is ——— 866
Sum of these is ——— 2598
Difference of the Diameter is 10, this Squared is 100, the half is ——— 50
The Remainder is ——— 2548

This Multiplied by 48, gives 122304, and this Divided by 2077 quotes 113,55, the Content in Ale-Gallons; or by 882 the Quotient will be 138.66, the Content in Wine-Gallons.

Thus it appears by the four Problems aforegoing, if the Dimensions of a Cask

Be { Bung Diameter ——— 34 / Head Diameter ——— 24 / Length ——— 48 } Inches.

The Content in each variety is as followeth.

		Ale-Gall.	Diff.
By Probl.	I.	128.71	5.17
	II.	123.54	5.17
	III.	118.37	4.82
	IV.	113.55	

And here it may be observed that the difference between the Content of the first and second, is equal to the difference between the second and third, and the difference between the third and fourth, is not above three Pints less than the other: so that by these

theſe Rules there is not ſo much room for error, as is by the common Method which ſeveral Authors have preſcribed, for by their Rules the Content of this Cask in the ſecond Notion would be but one Gallon leſs than in the firſt, and the Content in the third Notion but little above two Gallons more than in the fourth, and ſo the difference between the Second and third would be about 12 Gallons.

It may yet be ſuppoſed (and muſt be granted) that a Cask of theſe Diameters and Length, may contain ſome certain quantity between the Contents here found, as ſuppoſe 126, 121 or 116 Gallons.

In ſuch Caſes you may proceed by the Rule already given, adding 1, 2, 3, 4, or 5 tenth Parts of the Difference of the Squares, as your judgment and the ſhape of the Cask ſhall direct: and the like caution muſt be uſed in finding the mean Diameter, if you work by the Inſtrument. But when the difference of the Diameters of any Cask is but 4 or 5 Inches, the four Rules aforegoing (or indeed the three firſt of them) will anſwer all the varieties that can happen: as ſuppoſe the Dimenſions of a Cask be

Bung Diameter	28	Inches.
Head Diameter	24	
Length	40	

The Content in Wine-Gallons will be found to be as followeth,

		W.G.	Diff.
By Probl.	I.	97.22	1.88
	II.	95.34	1.88
	III.	93.46	1.31
	IV.	92.15	

I have

I have insisted the longer upon this business that I might prevent mistakes, for we cannot be too exact, where a small error must needs be prejudicial either to the King, or to the Merchant, especially when the Duty upon Brandy is great.

Problem V.

The Dimension of any Cask being given, to find the Content by the Instrument.

TO do this we must first find a mean Diameter, this is, such a Diameter as shall reduce the Cask to a Cylinder, for which purpose I have Calculated these three Tables, for the three first varieties of Casks abovementioned.

A Table for Equating the Diameters of the middle Frustum of a Spheroid.

	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.73	0.80	0.87	0.94	1.01	1.08	1.15	1.22	1.29
2	1.36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1.99
3	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
4	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
5	3.46	3.54	3.61	3.68	3.75	3.82	3.89	3.96	4.03	4.10
6	4.17	4.24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4.81
7	4.88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
8	5.60	5.68	5.75	5.82	5.90	5.97	6.04	6.11	6.18	6.25
9	6.32	6.40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
10	7.05	7.13	7.20	7.28	7.35	7.42	7.49	7.56	7.64	7.71

A Table for the Second Variety of Casks

	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.62	0.68	0.74	0.81	0.87	0.93	0.99	1.06	1.12	1.18
2	1.24	1.30	1.36	1.43	1.49	1.55	1.62	1.68	1.74	1.80
3	1.86	1.93	1.99	2.05	2.12	2.18	2.24	2.30	2.37	2.43
4	2.49	2.55	2.62	2.68	2.75	2.81	2.88	2.94	3.01	3.07
5	3.13	3.20	3.26	3.33	3.39	3.46	3.52	3.59	3.65	3.72
6	3.78	3.85	3.91	3.98	4.04	4.11	4.18	4.24	4.31	4.37
7	4.44	4.50	4.57	4.63	4.70	4.76	4.83	4.90	4.96	5.03
8	5.10	5.16	5.23	5.30	5.36	5.43	5.49	5.56	5.63	5.70
9	5.77	5.84	5.91	5.97	6.03	6.10	6.17	6.24	6.30	6.37
10	6.44	6.51	6.57	6.64	6.71	6.77	6.83	6.90	6.96	7.02

A Table for the Third Variety of Casks.

	0	.1	.2	.3	.4	.5	.6	.7	8.	.9
1	0.55	0.60	0.65	0.70	0.76	0.81	0.87	0.93	0.99	1.05
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.55	1.61
3	1.67	1.73	1.78	1.84	1.90	1.95	2.01	2.06	2.12	2.18
4	2.24	2.30	2.35	2.41	2.46	2.52	2.57	2.63	2.69	2.75
5	2.81	2.87	2.93	2.98	3.04	3.10	3.16	3.21	3.27	3.33
6	3.39	3.45	3.51	3.56	3.62	3.68	3.73	3.80	3.85	3.91
7	3.97	4.02	4.08	4.14	4.20	4.26	4.32	4.38	4.44	4.50
8	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98	5.04	5.10
9	5.16	5.22	5.28	5.34	5.41	5.47	5.53	5.59	5.65	5.71
10	5.77	5.83	5.89	5.96	6.02	6.08	6.14	6.20	6.26	6.32

Now by these Tables the mean Diameter of any Cask is thus found.

Subtract the Head Diameter from the Bung Diameter, then seek the difference in the Table, taking the whole Inches in the first Colum, and the tenths (if any be) at the Head, in the Angle of meeting you have a Number, which being added to the Head Diameter, gives the mean Diameter sought.

Example.

Let the Dimensions be as in the first Problem of this Section, the difference of the Diameter is 10. In the Table for a Spheroid, against 10 on the side, and under 0, I find 7.05, this added to the Head Diameter (*viz.* 24) the Sum is 31.05. the mean Diameter sought.

This being obtained, the Content of any Cask may be found by the Instrument at one Operation, for which the Proportion is:

As the Gage-Point is to the Casks length,
So is the mean Diameter to the Content.

Example.

The length 48, and the mean Diameter 31.05, Inches.

Set the Gage-Point for Ale-Gallons to 48 upon C, then against 31.05 upon D, is 128.71 the Content in Ale-Gallons, which is the same as was found by the first Problem of this Section.

Note, The three Tables above are expressed in three Scales, and placed upon one side of the Rule against the Line of Inches, by which the mean Diameter of any Cask may be readily found.

To find the Ullage of a Cask.

Problem VI.

Let Figure 21. represent a Cask posited with its Axis e f, Parallel to the Horizon, in part empty the Surface of the Liquor cutting the Bung Diameter in S, suppose the whole Content 128.71 Gallons, the Bung Diameter a b 34 Inches, the wet part s b 26, the dry part S a 8; How many Gallons are contained in the Cask? and how many will it require to fill it full.

FOR resolving this, and such like Questions, there is a Line of Segments on the Rule, with a Line of Numbers to slide against it, and upon these Lines the proportion is;

1. *As the Bung Diameter upon the Numbers, is to* 100 *upon the Segments,*

So is the wet or dry parts on the Numbers, to a fourth Number on the Segments.

2. *As* 100 *upon B, is to the Casks whole Content upon A,*

So is the fourth Number last found to the Answer.

Example.

Example.

1. Set 34 the Bung Diameter on the Numbers, to 100 on the Segments, then againſt 8 the dry Inches on the Numbers is 16 on the Segments.

2. Set 100 upon B, to 128.71 the Casks whole Content upon A, then againſt 16 upon B, you have 20.6 upon A, and ſo many Gallons it will take to fill up the Cask, this Subtracted from 128.71, the whole Content, there reſts 108.11 the quantity of Liquor in the Cask; which may otherwiſe be found thus, having ſet 34 the Bung Diameter to 100 on the Segments, againſt 26 the wet Inches on the Numbers, you have 84 on the Segments.

Then,

As 100 upon B, is to 128.71 upon A,
So is 84 upon B, to 108.11 upon A:
Which being the ſame as before is a proof of the Work.

Note, The Line of Segments on this Rule is made for a Spheroid, and is therefore more exact for all Bulging Casks than the Tables of Segments, which are proper only for a Cylinder.

The following Tables ſhew the Ullage of two ſeveral Casks in Gallons and Quarts: they were made by drawing out the Liquor by an exact Wine Quart, and the dry Inches taken by an Inſtrument proper for that purpoſe.

A Table shewing the Ullage of a Cask in Gallons and Quarts, the Length of the Cask is 37·3 Inches, the Bung Diameter 26·9, the Head Diameter 25, and the Content 100 Wine-Gallons.

Dry Inches	Gallons	Quarts
0.71		1
1.10		2
1.40		3
1.62	1	0
1.82	1	1
2.00	1	2
2.16	1	3
2.30	2	0
2.44	2	1
2.57	2	2
2.69	2	3
2.80	3	0
2.91	3	1
3.01	3	2
3.11	3	3
3.20	4	0
3.30	4	1
3.39	4	2
3.48	4	3
3.57	5	0
3.65	5	1
3.73	5	2
3.81	5	3
3.89	6	0
4.05	6	2
4.21	7	0
4.37	7	2
4.53	8	0
4.69	8	2
4.84	9	0
4.99	9	2
5.15	10	0
5.30	10	2
5.45	11	0
5.60	11	2
5.75	12	0
5.90	12	2
6.04	13	0

Dry Inches.	Gallons	Quarts.
6.19	13	2
6.33	14	0
6.48	14	2
6.62	15	0
6.76	15	2
6.90	16	0
7.04	16	2
7.18	17	0
7.32	17	2
7.46	18	0
7.60	18	2
7.73	19	0
7.87	19	2
8.00	20	0
8.14	20	2
8.26	21	0
8.39	21	2
8.52	22	0
8.65	22	2
8.77	23	0
8.90	23	2
9.02	24	0
9.14	24	2
9.26	25	0
9.38	25	2
9.50	26	0
9.62	26	2
9.73	27	0
9.85	27	2
9.96	28	0
10.08	28	2
10.19	29	0
10.31	29	2
10.42	30	0
10.54	30	2
10.65	31	0
10.76	31	2
10.87	32	0

Dry Inches.	Gallons	Quarts.
10.98	32	2
11.09	33	0
11.20	33	2
11.31	34	0
11.42	34	2
11.53	35	0
11.64	35	2
11.75	36	0
11.86	36	2
11.97	37	0
12.08	37	2
12.19	38	0
12.30	38	2
12.41	39	0
12.52	39	2
12.63	40	0
12.74	40	2
12.85	41	0
12.96	41	2
13.07	42	0
13.18	42	2
13.28	43	0
13.39	43	2
13.49	44	0
13.60	44	2
13.70	45	0
13.81	45	2
13.92	46	0
14.02	46	2
14.12	47	0
14.23	47	2
14.33	48	0
14.44	48	2
14.54	49	0
14.65	49	2
14.75	50	0

A Table shewing the Ullage of a Cask to every half Gallon, the Length of the Cask is 30.8 Inches, the Bung Diameter 25.6, the Head Diameter 22.6, and the Content 64 Wine-Gallons.

Dry Inch.	Gall. and Quar	Dry Inch.	Gall. and Quar	Dry Inch	Gall. and Quar
.80	2	5.76	11 2	9.60	22 2
1.25	1 0	5.94	12 0	9.77	23 0
1.60	1 2	6.12	12 2	9.94	23 2
1.91	2 0	6.13	13 0	10.11	24 0
2.17	2 2	6.48	13 2	10.28	24 2
2.40	3 0	6.66	14 0	10.45	25 0
2.63	3 2	6.84	14 2	10.62	25 2
2.85	4 0	7.01	15 0	10.79	26 0
3.07	4 2	7.19	15 2	10.96	26 2
3.28	5 0	7.36	16 0	11.13	27 0
3.49	5 2	7.54	16 2	11.30	27 2
3.69	6 0	7.71	17 0	11.47	28 0
3.89	6 2	7.89	17 2	11.64	28 2
4.08	7 0	8.06	18 0	11.81	29 0
4.27	7 2	8.24	18 2	11.98	29 2
4.46	8 0	8.41	19 0	12.14	30 0
4.65	8 2	8.58	19 2	12.31	30 2
4.84	9 0	8.75	20 0	12.47	31 0
5.03	9 2	8.92	20 2	12.64	31 2
5.21	10 0	9.09	21 0	12.80	33 0
5.40	10 2	9.26	21 2		
5.58	11 0	9.43	22 0		

The

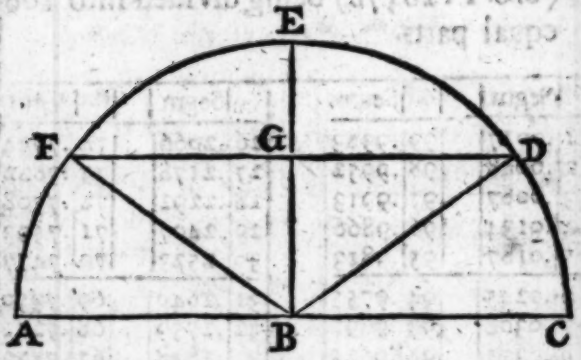
The Casks by which these Tables were made, were a Brandy Puntion, and a Brandy Hogshead, both from *Bourdeaux*: the Tables will serve indifferently for all such Casks, whose Diameters Length, and Content are the same with these, and are so plain that there needs no Example to shew their use; for against the dry Inches you have the Gallons and Quarts that the Cask wants of being full, and if the Bung Diameter be more than half dry, enter the Table with the wet Inches instead of the dry, it shews the quantity of Liquor in the Cask: These Tables may by proportion be converted into Lines, and put upon a Rule, as they have sometimes been by Mr. *Isaac Carver*, who is a very accurate Man at all such Operations.

By what hath been already said 'tis easie (I presume) to find the Ullage of a Cask whose Axis is posited Parallel to the Horizon: but lest this Tract should be thought imperfect, I shall here insert a Table of the Segments of a Circle, and also shew their Construction and Use.

A Table of the Segments of a Circle whose Area is Unity, the Diameter (*viz.* 1.128378) being divided into 100 equal parts.

Vs	Segm.	Vs	Segm.	Vs	Segm.	Vs	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	.7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7350
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0680	88	.9320	37	.3364	63	.6436
13	.0764	87	.9236	38	.3487	62	.6513
14	.0851	86	.9149	39	.3611	61	.6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Construction of the Table of Segments of a Circle.



In the Figure here before us, all that part or portion comprehended within the Right Lines D B and B F, and the Circular Line F E D is called the Sector of a Circle, F G D is the Cord of the Segment, G E is the Verſed Sine, and G B the Complement thereof, A B C is the Diameter of the Circle, which here we ſuppoſe to be 2.

Now if the Diameter of a Circle be 2, the length of the Circumference is double the Area of the Circle, for the Circumference is 6.2831852, and the Area 3.1415926; hence it evidently follows that the length of half the Arch of any Sector of a Circle, is equal to the Area of that Sector, and (as ſhall be proved anon) the Natural Sine of the ſaid half Arch is equal to the Area of the Triangle in that Sector.

Now the Area of the Triangle taken from the Area of the Sector leaves the Area of the Segment.

Example.

Example.

Let it be required to find the Area of the Segment F E D G, whoſe verſed Sine G E is .4.

1. *To find the Area of the Sector.*

In order to which we muſt firſt know the quantity of the Angle at the Center, and that is thus diſcovered.

Seek the Complement of the verſed Sine in the Table of Natural Sines, the Degrees and Decimal parts (or minutes) thereunto belonging are the Coſine of half the Angle at the Center.

Thus B E 1 — G E .4 = B G .6, this found in the Table of Natural Sines gives 36 Degrees and 85 Minutes, the Angle A B F, its Complement is 53 *d.* 13′. the Arch F E, which doubled is 106 *d.* 26′. the Arch of the Sector F E D, that is, the quantity of the Angle at the Centers.

Now to find the length of any number of Degrees, (that is the length of any Arch) of this Circle. Say,

As 360 *the Degrees in the whole Circle, to* 6.2831852 *the length of the whole Circumferene;*

So *is* 1, *to* .01745329 *the length of one Degree.*

And this is a common and fixed Multiplicator for any Number of Degrees, ſo the Degrees in the Arch F E, *viz.* 53.13, being Multiplied by .01745329 gives .92727787 the length of the Arch F E, that is, the Area of the Sector B F E D, which was required.

2. *To find the Area of the Triangle B F D.*

The Diameter being 2, I ſay half the Natural Sine of the Angle at the Center, when the ſaid Angle does not exceed 90 *d.* when it does then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus in the Sector B F E D, the Angle at the Center is 106.26 its Complement to 180. is 73.74, whoſe Natural Sine by the Table is .96 the half of this is .48 the Area of the Triangle B F D which was required.

The truth of which doth thus appear, B F is 1. B G is .6 *ergo* G F is .8 for B F q — B G q = F G q (by 47.1 of *Euclid.*)

But F G .8 Multiplied by B G .6 gives .48 the Area of the Triangle B F D as before, Then

From the Area of the Sector B F E D — .92727787
Take the Area of the Triangle B F D — .48000000

There remains the Area of the Segment F E D G — .44727787

But ſeeing in all Tables ſhewing the Areas of the Segments of a Circle it is ſuppoſed the Area of the whole Circle is Unity, the foregoing Table was made by this proportion.

As the Area of that Circle whoſe Diameter is 2, *viz.* 3.1415926, is to the Segment of any part of that Diameter:

So is 1 (the ſuppoſed Area of another Circle) to the Segment of the like part of its Diameter. Thus

As 3.1415926, is to .4472778 the Segment laſt found,

So

So is 1 to .14237, the Segment answering to the verſed Sine 20 in the Table,

For as the verſed Sine of the firſt Segment, *viz.* G E was .4 that is $\frac{2}{5}$ of the Radius B E:

So 20 the verſed Sine of this laſt Segment is $\frac{2}{5}$. of the Radius of the Table.

The Uſe of the Table of Segments

Is to find the Ullage or Quantity of Liquor remaining in a Cask, whoſe Axis is poſited Parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask, and for this the Rule is;

To the wet or dry Inches of the Bung Diameter add a competent number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under the Title V S gives a Segment, which Multiplied by the whole Content of the Cask, the Product ſhews the quantity of Liquor in the Cask, if the dividend were the wet Inches, or the Ullage if it were the dry.

Example.

Let there be a Cask in the form of a Cylinder, whoſe Bung Diameter is 33 Inches, the dry part 12, and the Content of the Cask 108 Gallons, how many Gallons are wanting to fill up the Cask?

12 Divided by 33 quotes .36, which found in the Table under V S gives .2341, this Segment Multiplied by 108 (the whole Content of the Cask) the Product is 35, and ſo many Gallons the Cask wants of being full.

Note, If a Cask be in the form of a Cylinder, or near that Figure, the Table will give the Ullage more

exactly than the Line of Segments on the Rule: but if the Bung Diameter be much greater than the Head Diameter, the Line of Segments is truer than the Table.

Problem VII. Figure 23.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, ſtanding upon one Head, with its Axis Perpendicular to the Horizon, the Dimenſions

Are { E H Length ——— 24 / A B Bung Diameter — 20 / C D Head Diameter — 16 } Inches.

To Calculate any Diameter betwixt the Bung and Head (ſuppoſe S V) and by ſuch Diameter to find the quantity of Liquor in the Cask when part is empty.

1. *To find the Diameter the Rule is,*

From the Square of the Bung Diameter (A B) Subtract the Square of the Head Diameter (C D) and Divide the Square-Root of the remainder by half the Casks length; (E O) then Multiply this Quotient by the Number of Inches, that the Diameter ſought is diſtant from the Bung, and call this Product your Subducend. Laſtly, from the Square of the Bung Diameter, Subtract the Square of your Subducend, the Square-Root of the remainder is the Diameter ſought.

Example

Example.

Let it be required to find the Diameter S V, which is ſuppoſed to be 6 Inches diſtant from A B the Bung Diameter.

The Square of the	Bung Diameter is	400
	Head Diameter is	256
The remainder is		144

Whoſe Square-Root is 12, this Divided by half the Casks Length, *viz.* 12, the Quotient is 1, this Multiplied by 6, gives 6 for the Subducend.

Square of the Bung Diameter	400
Square of the Subducend	36
There remains	364

Which is the Square of the Diameter S V, its Square Root is 1907878 the Diameter it ſelf, but the Square of it is ſufficient for our purpoſe.

2. *To find the Quantity of Liquor in the Cask.*

Having found the Square of S V, the Diameter of the Liquors Surface, the quantity of Liquor is eaſily attained:

For by the Rule laid down in the firſt Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale-Gallons, the half of this is 11.766, for the Content of the lower half, A n t B, and by the Rule now cited, the Content of the part A S V B will be found to be 6.484 Gallons.

For the Square of A B is	400
The Square of S V is	364
Sum is	764
Half Sum is	382
Half the difference of the Squares is	18
Sum of these is	1164

This Multiplied by *o g*, *viz.* 6, gives 6984, and this Divided by 1077 quotes 6.484, the Content of the part A S V B, as above; now this added to the Content of the lower half, *viz.* 11.766, gives 18.25, and so many Gallons are in the Cask, if this be taken from 23.532, the whole Content, there rests 5.282, and so many Gallons are required to fill up the Cask, for this 5.282 Gallons, is the Content of the part S C D V.

If the Cask were less than half full, the Ullage might be found by the same method, and that Subtracted from the whole Content leaves the quantity of Liquor in the Cask.

SECT.

SECT. X.

Of Gauging of Worts in open Veſſels.

FOR this purpoſe there are two Lines placed on the under ſide of the two Sliding Pieces of the ſhort Rule, which being ſo plain and obvious as not to need any Deſcription, I ſhall proceed to ſhew their Uſe, which is to find how many Ale-Gallons, and Hundred parts of of a Gallon, any ſmall Tub or ſuch like open Veſſel (from 12 to 36 Inches Diameter) will contain at one Inch deep, to perform which, obſerve the following Directions.

☞ Let the Sliding Piece C D, be pin'd faſt at the end towards the Right-Hand, then holding the Rule Parallel, draw out the other Sliding Piece 'till the two ends touch any two oppoſite ſides of the Tub or Veſſel, this done look what diviſions (both upon the Line of Gallons and Line of Inches upon the Sliding Piece) are cut by the end of the Rule, for that point doth ſhew the Diameter of the Tub in Inches and tenth parts upon one Line, and upon the other how many Gallons and Hundred parts it will contain at one Inch deep.

Thus when the end of the Rule cuts 13.4 on the Line of Inches, it will alſo cut 0 Gallons 5 tenths on the Line of Gallons; in like manner, when it cuts 21 on the Inch Line, it will at the ſame time cut 1.23 on the Line of Gallons; that is, 1 Gallon and .23 Hundred parts, and ſo much will a Tub

of 21 Inches Diameter contain at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half, and then the end of Rule will cut 1.673 on the Line of Gallons.

Now if the Diameter of the Tub be more than 24.5, pin this Piece at 24.5, and then draw out the other Sliding Piece at the other end, where you will find that when the end of the Rule cuts 30 Inches, it will also cut 2.51 Gallons, and thus you may draw out this Piece to 36 Inches, where you will find 3 Gallons, 6 tenths and more: *Note*, In all these Cases 'tis the Area only which is sought, the Inches do also shew the Diameter, but this is to make the other more plain.

Now the Tubs or Vessels which Victuallers do commonly make use of for cooling their Worts, and working their Ale, are Generally Cylindrical or Conical, and in either of these the Bases are Circular or Elliptical.

First of such as are Cylinders or near that form.

If the Bases of a Cylindrical Tub be Circular, the Rule doth shew the Area or Content at one Inch deep by inspection, (as above) this Multiplied by the depth of the Liquor gives the Content.

For Instance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 tenths, and the depth of the Wort 7 Inches and an half:

Multiply

Multiply the Area	1.4
by the depth, *viz.*	7.5
	70
	98
The Product is	10.50

That is, 10 Gallons and an half, the Content ſought.

But this Content may be found without the Pen, for having the Area and the depth, you may Multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4 the Area upon A; then againſt 7.5 the depth upon B, you have 10.5 upon A, the Content as before.

If the Veſſel be Elliptical, take the Croſs Diameters in the middle of the depth, as ſuppoſe the longeſt Diameter is 60 Inches, and the ſhorteſt 40; ſay,

As 359 is to one of the Diameters,

So is the other Diameter to the Area or Content.

And by this Proportion on the Lines A and B the Area of the Ellipſis above mentioned will be found to be 6.68: For,

As 359 upon B is to 40 upon A,

So is 60 upon B to 6,68 upon A.

This is the true way of finding the Area of an Ellipſis. But in ſmall Elliptical Tubs you may find the Area with leſs trouble:

Thus,

Find (by drawing out the Rule) the Area of the longeſt Diameter, and in like manner the Area of the ſhorteſt Diameter, half the Sum of theſe two is the Area ſought,

For Inſtance.

Suppoſe the Area of the longeſt Diameter be—3.62
And the Area of the ſhorteſt———2.51
The Sum is———6.13
The half is———3.065

The Area required; which is very near the truth, for if you Work by the former Rule, you will find the true Area to be but 3.008.

Secondly for Conical Veſſels.

When the Diameters at top and bottom differ not above 4 Inches, you may find the Area in the middle of the depth by drawing out the Rule as above directed, and this Area thus found is very near a true mean for the whole Tub.

Example.

Let the Diameter at the bottom of a Conical Tub be 24 Inches, and the Diameter at the Top 28 Inches, in the middle of the depth of this Tub you will find (by the Rule) the Diameter 26 Inches, and the Area 1.88 Gallons.

And for ſuch Conical Tubs (or Tuns) whoſe Diameters are above 36 Inches, if they be near a Cylinder one mean Inch will be ſufficient.

But for ſuch as are much Conical and great, you may find the whole Content by Problem V. Sect. VII. And if you deſire to Inch any ſuch Tun, obſerve the directions in Problem I. Sect. VIII.

There

There are yet another ſort of Veſſels which Victualers do frequently uſe to work their Ale in, and theſe are part of a But or Pipe ſet upon one Head, the other being cut off.

The whole Content of theſe may be found by Problem VII. Section IX. And how to Inch ſuch Veſſels ſhall be ſhewn in the Appendix.

I have obſerved in ſeveral places they Cool their Worts in Braſs-Pans whoſe Diameter is about 30 Inches, and depth 10 or 12 Inches, but theſe are commonly deeper in the middle by an Inch or two than at the ſide.

In ſuch Veſſels as theſe if you take (with the Rule) the Area of the Liquors Surface, and Multiply it by the depth taken within 3 or 4 Inches of the ſide, the Product will be the Content very near the truth.

And for Round Bowls which are like the Segment of a Globe, if you Multiply the Area of the Liquors Surface by half the depth, it gives the Content near enough in Practice: But in ſuch Caſes as theſe, Experience will be your beſt Tutor.

But that I may not be wanting to the meaneſt Capacities; I have here inſerted a very uſeful Table, ſhewing the Area's of Circles and Contents of Cylinders in Ale-Gallons, to all Diameters in Inches and tenths, from 12 to 156 Inches Diameter, and at any depth whatſoever.

The Table explained.

The two firſt Pages (and ſo every two next ſucceeding) contain 10 Columns, in the firſt of which you have the Diameters of Circles, beginning at 12, and continued down (in the firſt Column of each left-hand Page) in Inches and tenths to 156 Inches.

The

The other 9 Columns are Numbered at the to 1, 2, 3, 4, 5, 6, 7, 8, 9. in the firſt of theſe yo have the Area's of Circles at one Inch deep, to th ten thouſandth part of an Ale-Gallon.

In the next Column under 2, is the Content at Inches deep, under 3 you have the Content at Inches deep, in the next at 4 Inches deep; in lik manner under 5, is the Content at 5 Inches deep and ſo on to 9 Inches deep, in the laſt Column under 9.

The Uſes of this Table are,

1. *To find the Area of any Circle, the Diameter being given.*

And this it doth by inſpection, for in the Column under 1, you have the Area to any Diameter from 12 to 156: thus againſt this Diameter 43 in the firſt Column you have 5.1496 in the next Column, which is the Area ſought, the like for any other.

2. *To find the Content of any Cylindrical Tun, the Diameter and depth being given.*

To effect this, obſerve, when the depth is whole Inches only, and leſs than 10, you have the anſwer at once by inſpection, for againſt the Diameter, and under the depth is the Content ſought.

Example.

Let the Diameter of a Cylindrical Tun be 58.5 Inches, and the depth 6 Inches: What is the Content?

Seek 58 in the firſt Column, and in the fifth Line under it againſt 5 tenths, and under 6 (the depth) you

you will find 57.19, which is the Content ſought.

In like manner in the ſame Line under 7, you have 66.72, the Content at 7 Inches deep.

Note, If this Tun were 70 Inches deep the ſame Number, *viz.* 66.72 gives the Content, removing the prick one place towards the right Hand, and then it will be 667.2 Gallons.

And if the depth were but 7 tenth parts of an Inch, you muſt remove the prick one place towards the left Hand, and then it will be 6.672.

Hence 'tis evident this Table being made to 9 Inches deep, will ſhew the Content at any other depth whatſoever.

Example.

There is a Tun in the form of a Cylinder, the Diameter is 61 Inches, and the depth 35.6 Inches: What is the Content?

Find 61 in the firſt Column, and in the Line againſt it, under 3 you have 31.09, which by removing the prick is 310.9 the Content at 30 Inches deep, ſet this down as in the Margin.

Again,

In the ſame Line (againſt 61, and) under 5 is 51.81, ſet this down under the former, then for the 6 tenths take the Number under 6, (*viz.* 62.18) and removing the prick one place towards the left Hand ſet it under the other, theſe being added together:

30	——	310.9
5	——	51.81
0.6	——	6.218
The Sum is ——	35.6 =	368.928

which is the Content sought: and after the same manner you may find that a Tun of 61 Inches Diameter, and 28.5 Inches deep will contain 295.38 Gallons: See the Work:

20	——	207.3
8	——	82.9
0.5	——	5.18
28.5	=	295.38

A

TABLE

OF THE

AREA's of CIRCLES

And Contents of

CYLINDERS

IN

ALE-GALLONS,

To all Diameters (in Inches and Tenths) from twelve Inches to one hundred fifty and six.

Diam. in Inches.	DEPTH.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.65
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.93	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.48	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH.				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.48
2.50	3.00	3.50	4.00	4.50
2.53	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

Diam. in Inches.	DEPTH.			
	1	2	3	4
15.0	0.6266	1.25	1.88	2.50
.1	0.6350	1.27	1.91	2.54
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6605	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH.				
5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.55	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

Diam. in Inches.	DEPTH. 1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.88	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1029	2.21	3.31	4.41
20.0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86

DEPTH.				
5	6	7	8	9
4.51	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.85	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.64	8.74	9.83
5.51	6.62	7.72	8.82	9.93
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
21.0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.75	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.93	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22.0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.19	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23.0	1.4733	2.95	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.01
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.61	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

DEPTH.				
5	6	7	8	9
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.68	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.05	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.92
7.24	8.68	10.13	11.58	13.02
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	13.96
7.82	9.39	10.95	12.51	14.08
7.89	9.46	11.04	12.62	14.20
7.95	9.54	11.13	12.72	14.31

Diam. in Inches.	DEPTH.			
	1	2	3	4
24.0	1.6042	3.21	4.81	6.42
.1	1.6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6.63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25.0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26.0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06

DEPTH.				
5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.55
8.16	9.79	11.42	13.05	14.68
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
8.36	10.03	11.70	13.37	15.04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.61	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.69
9.34	11.21	13.08	14.94	16.81
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
10.00	12.00	14.00	16.00	18.00
10.08	12.09	14.11	16.12	18.14

Diam. in Inches.	DEPTH.			
	1	2	3	4
27.0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7.27	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96

DEPTH.				
5	6	7	8	9
10.15	12.18	14.21	16.24	18.27
10.23	12.27	14.32	16.36	18.41
10.30	12.36	14.42	16.48	18.54
10.38	12.45	14.53	16.61	18.68
10.45	12.55	14.64	16.73	18.82
10.53	12.64	14.74	16.85	18.95
10.61	12.73	14.85	16.97	19.09
10.68	12.82	14.96	17.10	19.23
10.76	12.91	15.06	17.22	19.37
10.84	13.01	15.18	17.34	19.51
10.92	13.10	15.28	17.46	19.65
11.00	13.19	15.39	17.59	19.79
11.08	13.29	15.50	17.71	19.93
11.15	13.38	15.61	17.84	20.27
11.23	13.48	15.72	17.97	20.22
11.31	13.57	15.83	18.10	20.36
11.39	13.67	15.95	18.22	20.50
11.47	13.76	16.06	18.35	20.65
11.55	13.86	16.17	18.48	20.79
11.63	13.96	16.28	18.61	20.93
11.71	14.05	16.39	18.74	21.08
11.79	14.15	16.51	18.86	21.22
11.87	14.25	16.62	18.99	21.37
11.96	14.35	16.74	19.13	21.52
12.04	14.44	16.85	19.26	21.66
12.12	14.54	16.96	19.39	21.81
12.20	14.64	17.08	19.52	21.96
12.28	14.74	17.19	19.65	22.10
12.37	14.84	17.31	19.78	22.26
12.45	14.94	17.43	19.92	22.41

Diam. in Inches.	DEPTH.			
	1	2	3	4
30.0	2.3066	5.01	7.52	10.03
.1	2.3233	5.05	7.57	10.09
.2	2.3401	5.08	7.62	10.16
.3	2.3570	5.11	7.67	10.23
.4	2.3739	5.15	7.72	10.29
.5	2.3908	5.18	7.77	10.36
.6	2.4078	5.21	7.82	10.43
.7	2.4249	5.25	7.87	10.50
.8	2.4420	5.28	7.93	10.57
.9	2.4592	5.32	7.98	10.64
31.0	2.4764	5.35	8.03	10.71
.1	2.4938	5.39	8.08	10.78
.2	2.5111	5.42	8.13	10.84
.3	2.5285	5.46	8.19	10.91
.4	2.5460	5.49	8.24	10.98
.5	2.5635	5.53	8.29	11.05
.6	2.5811	5.56	8.34	11.12
.7	2.5987	5.60	8.40	11.19
.8	2.6163	5.63	8.45	11.27
.9	2.6341	5.67	8.50	11.34
32.0	2.6519	5.70	8.56	11.41
.1	2.6698	5.74	8.61	11.48
.2	2.6877	5.78	8.66	11.55
.3	2.7057	5.81	8.72	11.62
.4	2.7237	5.85	8.77	11.69
.5	2.7418	5.88	8.83	11.77
.6	2.7599	5.92	8.88	11.84
.7	2.7781	5.96	8.93	11.91
.8	2.7963	5.99	8.99	11.98
.9	3.0146	6.03	9.04	12.06

DEPTH.				
5	6	7	8	9
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.18
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51
14.26	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13

Diam. in Inches.	DEPTH.			
	1	2	3	4
33.0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.15	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.18	9.26	12.35
.4	3.1069	6.21	9.32	12.43
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6.33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
34.0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13.18
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
35.0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
.5	3.5099	7.02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5605	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

DEPTH.				
5	6	7	8	9
15.17	18.20	21.23	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.79
15.53	18.64	21.75	24.85	27.96
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	23.07	26.36	29.66
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	27.14	30.53
17.06	25.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.76
17.75	21.30	24.85	28.39	31.94
17.85	21.42	24.99	28.55	32.13
17.95	21.53	25.12	28.71	32.30

Diam. in Inches.	DEPTH.			
	1	2	3	4
36.0	3.6095	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	8.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

DEPTH.				
5	6	7	8	9
18.05	21.66	25.27	28.87	32.48
18.15	21.78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18.65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.36	28.43	32.49	36.56
20.43	24.51	28.60	32.68	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93

Diam. in Inches.	DEPTH.			
	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2598	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

DEPTH.				
5	6	7	8	9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.18	40.71
22.73	27.28	31.82	36.37	40.91
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.63	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	29.34	34.23	39.11	44.00

Diam. in Inches.	DEPTH			
	1	2	3	4
42.0	4.9129	9.83	14.74	19.65
.1	4.9363	9.87	14.81	19.74
.2	4.9598	9.92	14.88	19.84
.3	4.9834	9.97	14.95	19.93
.4	5.0069	10.01	15.02	20.03
.5	5.0306	10.06	15.09	20.12
.6	5.0543	10.11	15.16	20.22
.7	5.0780	10.16	15.23	20.31
.8	5.1019	10.20	15.31	20.41
.9	5.1257	10.25	15.38	20.50
43.0	5.1496	10.30	15.45	20.60
.1	5.1736	10.35	15.52	20.69
.2	5.1977	10.40	15.59	20.79
.3	5.2218	10.44	15.67	20.89
.4	5.2459	10.49	15.74	20.99
.5	5.2701	10.54	15.81	21.08
.6	5.2944	10.59	15.88	21.18
.7	5.3187	10.64	15.96	21.28
.8	5.3430	10.69	16.03	21.37
.9	5.3675	10.74	16.10	21.47
44.0	5.3920	10.78	16.18	21.57
.1	5.4165	10.83	16.25	21.67
.2	5.4411	10.88	16.32	21.76
.3	5.4657	10.93	16.40	21.86
.4	5.4904	10.98	16.47	21.96
.5	5.5152	11.03	16.55	22.06
.6	5.5400	11.08	16.62	22.16
.7	5.5649	11.13	16.70	22.26
.8	5.5898	11.18	16.77	22.36
.9	5.6148	11.23	16.85	22.46

DEPTH.				
5	6	7	8	9
24.37	29.48	34.39	39.30	44.22
24.68	29.62	34.55	39.49	44.43
24.80	29.76	34.72	39.68	44.64
24.92	29.90	34.88	39.86	44.85
25.04	30.04	35.05	40.06	45.06
25.16	30.19	35.22	40.25	45.28
25.27	30.33	35.38	40.43	45.49
25.39	30.47	35.55	40.62	45.70
25.51	30.61	35.72	40.82	45.92
25.63	30.76	35.88	41.01	46.14
25.75	30.90	36.05	41.19	46.34
25.87	31.04	36.21	41.39	46.56
25.99	31.19	36.39	41.59	46.78
26.11	31.33	36.56	41.78	47.00
26.23	31.48	36.72	41.97	47.22
26.35	31.62	36.89	42.16	47.43
26.47	31.77	37.06	42.35	47.65
26.59	31.91	37.23	42.55	47.87
26.72	32.06	37.40	42.75	48.09
26.84	32.20	37.57	42.94	48.31
26.96	32.35	37.74	43.14	48.53
27.08	32.50	37.91	43.33	48.75
27.21	32.65	37.09	43.53	48.97
27.33	32.80	38.26	43.73	49.19
27.45	32.94	38.43	43.92	49.41
27.58	33.09	38.61	44.12	49.64
27.70	33.24	38.78	44.32	49.86
27.83	33.39	38.96	44.52	50.09
27.95	33.54	39.13	44.72	50.31
28.08	33.69	39.31	44.92	50.54

Diam. in Inches.	DEPTH.			
	1	2	3	4
45.0	5.6398	11.28	16.92	22.56
.1	5.6649	11.33	17.00	22.66
.2	5.6901	11.38	17.07	22.76
.3	5.7153	11.43	17.15	22.86
.4	5.7405	11.48	17.22	22.96
.5	5.7659	11.53	17.30	23.06
.6	5.7912	11.58	17.37	23.16
.7	5.8167	11.63	17.45	23.27
.8	5.8421	11.68	17.53	23.37
.9	5.8677	11.74	17.60	23.47
46.0	5.8933	11.79	17.68	23.57
.1	5.9189	11.84	17.76	23.68
.2	5.9446	11.89	17.84	23.78
.3	5.9704	11.94	17.91	23.88
.4	5.9962	11.99	17.99	23.98
.5	6.0221	12.04	18.07	24.09
.6	6.0480	12.10	18.14	24.19
.7	6.0740	12.15	18.22	24.30
.8	6.1000	12.20	18.30	24.40
.9	6.1261	12.25	18.38	24.50
47.0	6.1523	12.30	18.46	24.61
.1	6.1735	12.36	18.54	24.71
.2	6.2048	12.41	18.61	24.82
.3	6.2311	12.46	18.69	24.92
.4	6.2575	12.52	18.78	25.03
.5	6.2839	12.57	18.85	25.14
.6	6.3104	12.62	18.93	25.24
.7	6.3369	12.67	19.01	25.35
.8	6.3635	12.73	19.09	25.45
.9	6.3902	12.78	19.17	25.56

DEPTH.				
5	6	7	8	9
28.20	33.84	39.48	45.12	50.76
28.33	33.99	39.66	45.32	50.99
28.45	34.14	39.83	45.52	51.21
28.58	34.29	40.01	45.72	51.44
28.70	34.44	40.18	45.92	51.66
28.83	34.60	40.36	46.13	51.90
28.96	34.75	40.54	46.33	52.12
29.09	34.90	40.72	46.54	52.35
29.21	35.05	40.90	46.74	52.58
29.34	35.21	41.08	46.94	52.81
29.47	35.36	41.25	47.15	53.04
29.60	35.51	41.43	47.35	53.27
29.73	35.67	41.62	47.56	53.51
29.85	35.82	41.79	47.76	53.73
29.98	35.98	41.97	47.97	53.97
30.11	36.13	42.15	48.18	54.20
30.24	36.29	42.34	48.38	54.43
30.37	36.44	42.52	48.59	54.67
30.50	36.60	42.70	48.80	54.90
30.63	36.76	42.88	49.01	55.13
30.76	36.91	43.07	49.22	55.37
30.89	37.07	43.25	49.43	55.60
31.03	37.23	43.44	49.64	55.85
31.16	37.39	43.62	49.85	56.08
31.29	37.55	43.81	50.07	56.32
31.42	37.71	43.99	50.27	56.56
31.55	37.86	44.17	50.48	56.79
31.69	38.02	44.36	50.70	57.03
31.82	38.18	44.54	50.91	57.27
31.95	38.34	44.73	51.12	57.51

Diam. in Inches.	DEPTH.			
	1	2	3	4
48.0	6.4169	12.85	19.25	25.67
.1	6.4436	12.89	19.33	25.78
.2	6.4705	12.94	19.41	25.88
.3	6.4973	12.99	19.49	25.99
.4	6.5243	13.05	19.57	26.10
.5	6.5513	13.10	19.65	26.20
.6	6.5783	13.16	19.74	26.31
.7	6.6054	13.21	19.82	26.42
.8	6.6325	13.27	19.90	26.53
.9	6.6600	13.32	19.98	26.64
49.0	6.6870	13.37	20.06	26.74
.1	6.7143	13.43	20.14	26.86
.2	6.7417	13.48	20.23	26.97
.3	6.7692	13.54	20.31	27.08
.4	6.7966	13.59	20.39	27.19
.5	6.8242	13.65	20.47	27.30
.6	6.8518	13.70	20.56	27.41
.7	6.8794	13.76	20.64	27.52
.8	6.9072	13.81	20.72	27.63
.9	6.9349	13.87	20.81	27.74
50.0	6.9628	13.93	20.89	27.85
.1	6.9906	13.98	20.97	27.96
.2	7.0186	14.04	21.06	28.08
.3	7.0466	14.09	21.14	28.19
.4	7.0746	14.15	21.22	28.30
.5	7.1027	14.21	21.31	28.41
.6	7.1309	14.26	21.39	28.52
.7	7.1591	14.32	21.48	28.64
.8	7.1873	14.38	21.56	28.75
.9	7.2157	14.43	21.65	28.86

DEPTH.				
5	6	7	8	9
32.08	38.50	44.92	51.34	57.75
32.22	38.66	45.11	51.55	58.00
32.35	38.82	45.29	51.76	58.23
32.49	38.98	45.48	51.98	58.48
32.62	39.15	45.67	52.19	58.72
32.76	39.31	45.86	52.41	58.96
32.89	39.47	46.05	52.63	59.20
33.03	39.63	46.24	52.84	59.45
33.16	39.79	46.43	53.06	59.69
33.30	39.96	46.62	53.28	59.94
33.43	40.12	46.80	53.49	60.18
33.57	40.29	47.00	53.71	60.43
33.71	40.45	47.20	53.94	60.68
33.85	40.61	47.38	54.15	60.92
33.99	40.78	47.58	54.38	61.17
34.12	40.95	47.77	54.59	61.42
34.26	41.11	47.97	54.82	61.67
34.40	41.28	48.15	55.03	61.91
34.54	41.44	48.35	55.26	62.16
34.68	41.61	48.55	55.48	62.42
34.82	41.78	48.74	55.71	62.66
34.96	41.95	48.94	55.93	62.92
35.10	42.11	49.13	56.15	63.17
35.24	42.28	49.33	56.38	63.42
35.38	42.45	49.53	56.60	63.68
35.52	42.62	49.72	56.83	63.93
35.66	42.79	49.92	57.05	64.18
35.80	42.96	50.11	57.27	64.43
35.94	43.12	50.31	57.50	64.69
36.08	43.30	50.51	57.73	64.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
51.0	7.2440	14.49	21.73	28.98
.1	7.2725	14.55	21.82	29.09
.2	7.3010	14.60	21.90	29.20
.3	7.3295	14.66	21.99	29.32
.4	7.3581	14.72	22.07	29.43
.5	7.3868	14.77	22.16	29.55
.6	7.4155	14.83	22.25	29.66
.7	7.4443	14.89	22.33	29.78
.8	7.4731	14.95	22.42	29.89
.9	7.5020	15.00	22.51	30.01
52.0	7.5309	15.06	22.59	30.12
.1	7.5599	15.12	22.68	30.24
.2	7.5890	15.18	22.77	30.36
.3	7.6181	15.24	22.86	30.47
.4	7.6472	15.29	22.94	30.59
.5	7.6764	15.35	23.03	30.71
.6	7.7057	15.41	23.12	30.83
.7	7.7350	15.47	23.21	30.94
.8	7.7644	15.53	23.29	31.06
.9	7.7939	15.59	23.38	31.18
53.0	7.8233	15.65	23.47	31.29
.1	7.8529	15.71	23.56	31.41
.2	7.8825	15.76	23.65	31.53
.3	7.9122	15.82	23.74	31.65
.4	7.9419	15.88	23.83	31.77
.5	7.9717	15.94	23.92	31.89
.6	8.0015	16.00	24.00	32.01
.7	8.0314	16.06	24.09	32.13
.8	8.0613	16.12	24.18	32.24
.9	8.0913	16.18	24.27	32.36

DEPTH.				
5	6	7	8	9
36.22	43.47	50.71	57.95	65.20
36.36	43.63	50.91	58.18	65.45
36.51	43.81	51.11	58.41	65.71
36.65	43.97	51.30	58.63	65.96
36.79	44.15	51.51	58.87	66.22
36.94	44.32	51.71	59.10	66.48
37.08	44.49	51.91	59.32	66.74
37.22	44.66	52.11	59.55	67.00
37.37	44.84	52.31	59.79	67.26
37.51	45.01	52.52	60.02	67.52
37.66	45.19	52.72	60.25	67.78
37.80	45.36	52.92	60.48	68.04
37.95	45.53	53.12	60.71	68.30
38.09	45.71	53.33	60.95	68.56
38.24	45.88	53.53	61.18	68.82
38.38	46.06	53.73	61.41	69.09
38.53	46.24	53.94	61.65	69.36
38.68	46.41	54.15	61.88	69.62
38.82	46.59	54.35	62.11	69.88
38.97	46.76	54.56	62.35	70.15
39.12	46.94	54.76	62.59	70.41
39.27	47.12	54.97	62.82	70.68
39.41	47.29	55.18	63.06	70.94
39.56	47.47	55.39	63.30	71.21
39.71	47.65	55.60	63.54	71.48
39.86	47.83	55.81	63.78	71.75
40.01	48.01	56.01	64.01	72.01
40.16	48.19	56.22	64.25	72.28
40.31	48.37	56.43	64.49	72.45
40.46	48.55	56.64	64.73	72.82

Diam. in Inches.	DEPTH.			
	1	2	3	4
54.0	8.1214	16.24	24.36	32.49
.1	8.1515	16.30	24.45	32.61
.2	8.1816	16.36	24.54	32.72
.3	8.2118	16.42	24.64	32.85
.4	8.2421	16.48	24.73	32.97
.5	8.2724	16.54	24.82	33.09
.6	8.3028	16.61	24.91	33.22
.7	8.3333	16.67	25.00	33.33
.8	8.3638	16.72	25.09	33.46
.9	8.3943	16.79	25.18	33.58
55.0	8.4249	16.85	25.27	33.70
.1	8.4556	16.91	25.37	33.82
.2	8.4863	16.97	25.46	33.95
.3	8.5171	17.03	25.55	34.07
.4	8.5479	17.10	25.64	34.19
.5	8.5788	17.16	25.74	34.32
.6	8.6097	17.22	25.83	34.44
.7	8.6407	17.28	25.92	34.56
.8	8.6718	17.34	26.01	34.68
.9	8.7029	17.40	26.11	34.81
56.0	8.7341	17.47	26.20	34.94
.1	8.7653	17.53	26.30	35.06
.2	8.7966	17.59	26.39	35.19
.3	8.8279	17.66	26.48	35.31
.4	8.8593	17.72	26.58	35.44
.5	8.8907	17.78	26.67	35.56
.6	8.9222	17.84	26.77	35.69
.7	8.9538	17.91	26.86	35.82
.8	8.9854	17.97	26.96	35.91
.9	9.9171	17.03	27.05	36.07

DEPTH.				
5	6	7	8	9
40.61	48.73	56.85	64.97	73.09
40.76	48.91	57.06	65.21	73.36
40.91	49.09	57.27	65.45	73.63
41.06	49.27	57.49	65.70	73.91
41.21	49.45	57.70	65.94	74.18
41.36	49.63	57.91	66.18	74.45
41.52	49.82	58.12	66.43	74.73
41.67	50.00	58.33	66.67	75.00
41.82	50.19	58.55	66.91	75.28
41.97	50.37	58.76	67.15	75.55
42.12	50.55	58.98	67.40	75.83
42.28	50.74	59.19	67.65	76.11
42.43	50.92	59.40	67.89	76.38
42.59	51.10	59.62	68.14	76.66
42.74	51.29	59.84	68.39	76.93
42.90	51.48	60.05	68.63	77.21
43.05	51.66	60.27	68.88	77.49
43.20	51.85	60.49	69.13	77.77
43.36	52.03	60.70	69.37	78.04
43.52	52.22	60.92	69.62	78.33
43.67	52.40	61.14	69.87	78.61
43.83	52.59	61.36	70.12	78.89
43.99	52.78	61.58	70.38	79.18
44.14	52.97	61.80	70.62	79.45
44.30	53.16	62.01	70.87	79.73
44.46	53.35	62.24	71.13	80.02
44.61	53.53	62.46	71.38	80.30
44.77	53.72	62.68	71.63	80.59
44.93	53.91	62.90	71.88	80.87
45.09	54.10	63.12	72.14	81.15

Diam. in Inches.	DEPTH.			
	1	2	3	4
57.0	9.0489	18.10	27.15	36.20
.1	9.0806	18.16	27.24	36.32
.2	9.1124	18.22	27.34	36.45
.3	9.1443	18.29	27.43	36.58
.4	9.1762	18.35	27.53	36.71
.5	9.2082	18.42	27.62	36.83
.6	9.2403	18.48	27.72	36.96
.7	9.2724	18.54	27.82	37.09
.8	9.3046	18.61	27.91	37.22
.9	9.3368	18.67	28.01	37.35
58.0	9.3691	18.74	28.11	37.48
.1	9.4014	18.80	28.20	37.60
.2	9.4338	18.87	28.30	37.74
.3	9.4662	18.93	28.40	37.86
.4	9.4988	19.00	28.50	38.00
.5	9.5313	19.06	28.59	38.12
.6	9.5639	19.13	28.69	38.26
.7	9.5966	19.19	28.79	38.38
.8	9.6293	19.26	28.89	38.52
.9	9.6621	19.32	28.99	38.65
59.0	9.6949	19.39	29.08	38.78
.1	9.7278	19.46	29.19	38.91
.2	9.7608	19.52	29.28	39.04
.3	9.7938	19.59	29.38	39.18
.4	9.8268	19.65	29.48	39.31
.5	9.8600	19.72	29.58	39.44
.6	9.8931	19.79	29.68	39.57
.7	9.9263	19.85	29.78	39.70
.8	9.9596	19.92	29.88	39.84
.9	9.9930	19.99	29.98	39.97

DEPTH.				
5	6	7	8	9
45.25	54.79	63.34	72.29	81.44
45.40	54.48	63.56	72.65	81.73
45.56	54.67	63.79	72.90	82.01
45.72	54.87	64.01	73.15	82.30
45.88	55.06	64.23	73.41	82.59
46.04	55.25	64.46	73.67	82.87
46.20	55.44	64.68	73.92	83.16
46.36	55.63	64.91	74.18	83.45
46.52	55.83	65.13	74.43	83.74
46.69	56.02	65.36	74.70	84.03
46.85	56.21	65.58	74.95	84.32
47.01	56.41	65.81	75.21	84.61
47.17	56.60	66.04	75.47	84.91
47.33	56.80	66.26	75.73	85.20
47.50	56.99	66.49	75.99	85.49
47.66	57.19	66.72	76.25	85.78
47.82	57.38	66.95	76.51	86.08
47.98	57.58	67.17	76.77	86.37
48.15	57.77	67.40	77.03	86.66
48.31	57.97	67.63	77.30	86.96
48.48	58.17	67.87	77.56	87.26
48.64	58.37	68.10	77.82	87.55
48.81	58.57	68.33	78.09	87.85
48.97	58.77	68.56	78.35	88.15
49.14	58.96	68.79	78.62	88.44
49.30	59.16	69.02	78.88	88.74
49.47	59.36	69.25	79.15	89.04
49.63	59.56	69.48	79.41	89.34
49.80	59.75	69.71	79.67	89.63
49.97	59.96	69.95	79.95	89.94

Diam in Inches.	DEPTH. 1	2	3	4
60.0	10.0264	20.05	30.08	40.10
.1	10.0598	20.12	30.18	40.24
.2	10.0933	20.19	30.28	40.37
.3	10.1269	20.25	30.38	40.51
.4	10.1605	20.32	30.48	40.64
.5	10.1942	20.39	30.58	40.78
.6	10.2279	20.46	30.68	40.91
.7	10.2617	20.52	30.78	41.04
.8	10.2955	20.59	30.89	41.18
.9	10.3294	20.66	30.99	41.32
61.0	10.3634	20.73	31.09	41.45
.1	10.3974	20.80	31.19	41.59
.2	10.4314	20.86	31.29	41.72
.3	10.4655	20.93	31.40	41.86
.4	10.4997	21.00	31.50	42.00
.5	10.5339	21.07	31.60	42.14
.6	10.5682	21.14	31.70	42.27
.7	10.6026	21.21	31.81	42.41
.8	10.6370	21.27	31.91	42.55
.9	10.6714	21.34	32.01	42.68
62.0	10.7059	21.41	32.11	42.82
.1	10.7405	21.48	32.22	42.96
.2	10.7751	21.55	32.33	43.10
.3	10.8098	21.62	32.43	43.24
.4	10.8445	21.69	32.53	43.38
.5	10.8793	21.76	32.64	43.52
.6	10.9141	21.83	32.74	43.66
.7	10.9490	21.90	32.85	43.80
.8	10.9840	21.97	32.95	43.94
.9	11.0190	22.04	33.06	44.08

DEPTH.				
5	6	7	8	9
50.13	60.16	70.18	80.21	90.23
50.30	60.36	70.42	80.48	90.54
50.47	60.56	70.65	80.75	90.84
50.64	60.76	70.89	81.02	91.14
50.80	60.96	71.12	81.28	91.45
50.97	61.17	71.36	81.55	91.75
51.14	61.37	71.60	81.82	92.05
51.31	61.57	71.83	82.09	92.35
51.48	61.77	72.07	82.36	92.66
51.65	61.97	72.30	82.63	92.96
51.81	62.18	72.54	82.90	93.27
51.99	62.38	72.78	83.18	93.58
52.16	62.59	73.02	83.45	93.88
52.33	62.79	73.26	83.72	94.19
52.50	63.00	73.50	84.00	94.50
52.67	63.20	73.74	84.27	94.81
52.84	63.41	73.98	84.55	95.11
53.02	63.62	74.22	84.82	95.43
53.19	63.82	74.46	85.10	95.73
53.36	64.03	74.70	85.37	96.04
53.53	64.24	74.94	85.65	96.35
53.70	64.44	75.18	85.92	96.66
53.88	64.65	75.43	86.20	96.98
54.05	64.86	75.67	86.48	97.29
54.22	65.06	75.91	86.75	97.60
54.40	65.27	76.15	87.03	97.91
54.57	65.48	76.40	87.31	98.23
54.75	65.69	76.64	87.59	98.54
54.92	65.91	76.89	87.87	98.86
55.10	66.11	77.13	88.15	99.17

Diam. in Inches.	DEPTH.			
	1	2	3	4
63.0	11.0541	22.11	33.16	44.22
.1	11.0892	22.18	33.27	44.36
.2	11.1244	22.25	33.37	44.50
.3	11.1596	22.32	33.48	44.64
.4	11.1949	22.39	33.59	44.78
.5	11.2302	22.46	33.69	44.92
.6	11.2656	22.53	33.79	45.06
.7	11.3011	22.60	33.90	45.20
.8	11.3366	22.67	34.01	45.35
.9	11.3721	22.74	34.12	45.49
64.0	11.4078	22.82	34.22	45.63
.1	11.4434	22.89	34.33	45.77
.2	11.4792	22.96	34.44	45.92
.3	11.5150	23.03	34.54	46.06
.4	11.5508	23.10	34.65	46.20
.5	11.5867	23.17	34.76	46.35
.6	11.6227	23.25	34.87	46.49
.7	11.6587	23.32	34.98	46.64
.8	11.6947	23.39	35.08	46.78
.9	11.7309	23.46	35.19	46.92
65.0	11.7670	23.53	35.30	47.07
.1	11.8033	23.60	35.40	47.21
.2	11.8396	23.68	35.52	47.36
.3	11.8759	23.75	35.62	47.50
.4	11.9123	23.82	35.74	47.65
.5	11.9488	23.90	35.85	47.80
.6	11.9853	23.97	35.96	47.94
.7	12.0219	24.04	36.07	48.09
.8	12.0585	24.12	36.17	48.23
.9	12.0952	24.19	36.28	48.38

DEPTH.				
5	6	7	8	9
55.27	66.32	77.38	88.43	99.49
55.45	66.53	77.62	88.71	99.80
55.62	66.75	77.87	88.99	100.12
55.80	66.96	78.12	89.28	100.44
55.98	67.17	78.37	89.56	100.76
56.15	67.38	78.61	89.84	101.07
56.33	67.60	78.86	90.13	101.40
56.51	67.81	79.11	90.41	101.71
56.69	68.02	79.36	90.70	102.03
56.86	68.23	79.60	90.98	102.35
57.04	68.45	79.86	91.26	102.67
57.22	68.66	80.10	91.54	102.99
57.40	68.87	80.35	91.83	103.31
57.57	69.09	80.61	92.12	103.64
57.75	69.31	80.86	92.41	103.96
57.94	69.52	81.11	92.70	104.28
58.12	69.74	81.36	92.98	104.61
58.30	69.95	81.61	93.27	104.93
58.47	70.17	81.87	93.56	105.26
58.65	70.39	82.12	93.85	105.58
58.84	70.60	82.37	94.14	105.90
59.01	70.81	82.62	94.42	106.22
59.20	71.04	82.88	94.72	106.56
59.38	71.26	83.13	95.01	106.88
59.56	71.47	83.38	95.30	107.21
59.75	71.69	83.64	95.60	107.54
59.93	71.91	83.90	95.88	107.87
60.11	72.13	84.15	96.18	108.20
60.29	72.35	84.41	96.46	108.52
60.48	72.57	84.67	96.76	108.86

Diam. in Inches.	DEPTH.			
	1	2	3	4
66.0	12.1319	24.26	36.40	48.53
.1	12.1687	24.34	36.51	48.68
.2	12.2055	24.41	36.62	48.82
.3	12.2424	24.48	36.73	48.97
.4	12.2794	24.56	36.84	49.12
.5	12.3164	24.63	36.95	49.27
.6	12.3535	24.71	37.06	49.41
.7	12.3906	24.78	37.17	49.56
.8	12.4278	24.86	37.28	49.71
.9	12.4650	24.93	37.40	49.86
67.0	12.5023	25.00	37.51	50.01
.1	12.5397	25.08	37.62	50.16
.2	12.5771	25.15	37.73	50.31
.3	12.6145	25.23	37.84	50.46
.4	12.6520	25.30	37.96	50.61
.5	12.6896	25.38	38.07	50.76
.6	12.7272	25.45	38.18	50.91
.7	12.7649	25.53	38.30	51.06
.8	12.8027	25.61	38.41	51.21
.9	12.8405	25.68	38.52	51.36
68.0	12.8783	25.76	38.63	51.51
.1	12.9162	25.83	38.75	51.66
.2	12.9542	25.91	38.86	51.82
.3	12.9922	25.98	38.98	51.97
.4	13.0303	26.06	39.09	52.12
.5	13.0684	26.14	39.20	52.27
.6	13.1066	26.21	39.32	52.42
.7	13.1448	26.29	39.44	52.58
.8	13.1831	26.37	39.55	52.73
.9	13.2215	26.44	39.66	52.88

DEPTH.				
5	6	7	8	9
60.66	72.79	84.92	97.06	109.19
60.85	73.01	85.18	97.35	109.32
61.03	73.23	85.44	97.64	109.85
61.21	73.45	85.69	97.94	110.18
61.40	73.67	85.95	98.23	110.51
61.58	73.90	86.21	98.53	110.84
61.77	74.12	86.47	98.83	111.18
61.95	74.34	86.73	99.12	111.51
62.14	74.57	87.00	99.42	111.85
62.33	74.79	87.26	99.72	112.19
62.51	75.01	87.51	100.02	112.52
62.70	75.24	87.78	100.32	112.86
62.89	75.46	88.04	100.62	113.19
63.07	75.68	88.30	100.91	113.53
63.26	75.91	88.56	101.22	113.87
63.45	76.13	88.82	101.51	114.20
63.64	76.36	89.09	101.82	114.54
63.83	76.59	89.36	102.12	114.89
64.02	76.82	89.62	102.42	115.23
64.20	77.04	89.88	102.72	115.56
64.39	77.27	90.14	103.02	115.90
64.58	77.50	90.41	103.33	116.24
64.77	77.72	90.68	103.63	116.59
64.96	77.95	90.94	103.94	116.93
65.15	78.18	91.21	104.24	117.27
65.34	78.41	91.48	104.54	117.61
65.53	78.64	91.74	104.85	117.95
65.73	78.87	92.02	105.16	118.31
65.92	79.10	92.28	105.47	118.65
66.11	79.33	92.55	105.77	118.99

Diam. in Inches.	DEPTH.			
	1	2	3	4
69.0	13.2599	26.52	39.78	53.04
.1	13.2983	26.60	39.89	53.19
.2	13.3368	26.67	40.01	53.35
.3	13.3754	26.75	40.13	53.50
.4	13.4140	26.83	40.24	53.66
.5	13.4527	26.91	40.36	53.81
.6	13.4915	26.98	40.47	53.96
.7	13.5303	27.06	40.59	54.12
.8	13.5691	27.14	40.71	54.28
.9	13.6080	27.22	40.82	54.43
70.0	13.6470	27.29	40.94	54.59
.1	13.6860	27.37	41.06	54.74
.2	13.7251	27.45	41.18	54.90
.3	13.7642	27.53	41.29	55.06
.4	13.8034	27.61	41.41	55.21
.5	13.8426	27.68	41.53	55.37
.6	13.8819	27.76	41.65	55.53
.7	13.9213	27.84	41.76	55.68
.8	13.9607	27.92	41.88	55.84
.9	14.0002	28.00	42.00	56.00
71.0	14.0397	28.08	42.12	56.16
.1	14.0793	28.16	42.24	56.32
.2	14.1189	28.24	42.36	56.48
.3	14.1586	28.32	42.47	56.63
.4	14.1983	28.40	42.59	56.79
.5	14.2381	28.48	42.71	56.95
.6	14.2780	28.56	42.83	57.11
.7	14.3179	28.64	42.95	57.27
.8	14.3579	28.72	43.07	57.43
.9	14.3979	28.80	43.19	57.59

DEPTH.				
5	6	7	8	9
66.30	79.56	92.82	106.08	119.34
66.49	79.79	93.09	106.38	119.68
66.69	80.02	93.36	106.70	120.03
66.88	80.25	93.63	107.00	120.38
67.07	80.48	93.90	107.31	120.73
67.27	80.72	94.17	107.62	121.08
67.46	80.95	94.44	107.93	121.42
67.65	81.18	94.71	108.24	121.77
67.85	81.41	94.98	108.55	122.12
68.04	81.65	95.26	108.86	122.47
68.24	81.88	95.53	109.18	122.82
68.43	82.12	95.80	109.49	123.17
68.63	82.35	96.08	109.80	123.53
68.82	82.58	96.35	110.11	123.88
69.02	82.82	96.62	110.42	124.23
69.21	83.05	96.89	110.74	124.58
69.41	83.29	97.17	111.06	124.94
69.61	83.53	97.45	111.37	125.29
69.81	83.77	97.73	111.69	125.65
70.00	84.00	98.00	112.00	126.00
70.20	84.24	98.28	112.32	126.36
70.40	84.47	98.55	112.63	126.71
70.60	84.71	98.83	112.95	127.07
70.79	84.95	99.11	113.26	127.42
70.99	85.19	99.39	113.58	127.78
71.19	85.43	99.67	113.90	128.14
71.39	85.67	99.95	114.22	128.50
71.59	85.91	100.23	114.54	128.86
71.79	86.15	100.51	114.86	129.22
71.99	86.30	100.79	115.18	129.58

Diam. in Inches.	DEPTH.			
	1	2	3	4
72.0	14.4380	28.88	43.31	57.75
.1	14.4781	28.96	43.43	57.91
.2	14.5183	29.04	43.55	58.07
.3	14.5585	29.12	43.67	58.23
.4	14.5988	29.20	43.80	58.40
.5	14.6392	29.28	43.92	58.56
.6	14.6796	29.36	44.04	58.72
.7	14.7201	29.44	44.16	58.88
.8	14.7606	29.52	44.28	59.04
.9	14.8012	29.60	44.40	59.20
73.0	14.8418	29.68	44.53	59.37
.1	14.8825	29.76	44.65	59.53
.2	14.9232	29.85	44.77	59.69
.3	14.9640	29.93	44.89	59.86
.4	15.0049	30.01	45.02	60.02
.5	15.0458	30.09	45.14	60.18
.6	15.0868	30.17	45.26	60.35
.7	15.1278	30.26	45.38	60.51
.8	15.1689	30.34	45.51	60.68
.9	15.2100	30.42	45.63	60.84
74.0	15.2512	30.50	45.75	61.00
.1	15.2925	30.58	45.88	61.17
.2	15.3338	30.67	46.00	61.34
.3	15.3751	30.75	46.13	61.50
.4	15.4165	30.83	46.25	61.66
.5	15.4580	30.92	46.37	61.83
.6	15.4950	30.99	46.49	61.98
.7	15.5411	31.08	46.62	62.16
.8	15.5827	31.17	46.75	62.33
.9	15.6244	31.25	46.87	62.50

DEPTH.				
5	6	7	8	9
72.19	86.63	101.07	115.50	129.94
72.39	86.87	101.35	115.82	130.30
72.59	87.11	101.63	116.14	130.66
72.79	87.35	101.91	116.46	131.02
73.00	87.59	102.19	116.79	131.39
73.20	87.83	102.47	117.11	131.75
73.40	88.07	102.75	117.43	132.11
73.60	88.32	103.04	117.76	132.48
73.80	88.56	103.32	118.08	132.84
74.01	88.81	103.61	118.41	133.21
74.21	89.05	103.89	118.74	133.58
74.41	89.29	104.17	119.06	133.94
74.62	89.54	104.46	119.38	134.31
74.82	89.78	104.75	119.71	134.68
75.03	90.03	105.04	120.04	135.05
75.23	90.28	105.32	120.37	135.41
75.44	90.52	105.61	120.70	135.78
75.64	90.77	105.90	121.02	136.15
75.85	91.01	106.18	121.35	136.52
76.05	91.26	106.47	121.68	136.89
76.26	91.51	106.76	122.01	137.26
76.46	91.75	107.04	122.34	137.63
76.67	92.00	107.34	122.67	138.01
76.88	92.25	107.63	123.00	138.38
77.08	92.50	107.91	123.33	138.74
77.29	92.75	108.21	123.66	139.12
77.48	92.97	108.47	123.96	139.46
77.71	93.25	108.79	124.33	139.87
77.92	93.50	109.08	124.66	140.25
78.12	93.74	109.37	124.99	140.62

Diam. in Inches.	DEPTH.			
	1	2	3	4
72.0	14.4380	28.88	43.31	57.75
.1	14.4781	28.96	43.43	57.91
.2	14.5183	29.04	43.55	58.07
.3	14.5585	29.12	43.67	58.23
.4	14.5988	29.20	43.80	58.40
.5	14.6392	29.28	43.92	58.56
.6	14.6796	29.36	44.04	58.72
.7	14.7201	29.44	44.16	58.88
.8	14.7606	29.52	44.28	59.04
.9	14.8012	29.60	44.40	59.20
73.0	14.8418	29.68	44.53	59.37
.1	14.8825	29.76	44.65	59.53
.2	14.9232	29.85	44.77	59.69
.3	14.9640	29.93	44.89	59.86
.4	15.0049	30.01	45.02	60.02
.5	15.0458	30.09	45.14	60.18
.6	15.0868	30.17	45.26	60.35
.7	15.1278	30.26	45.38	60.51
.8	15.1689	30.34	45.51	60.68
.9	15.2100	30.42	45.63	60.84
74.0	15.2512	30.50	45.75	61.00
.1	15.2925	30.58	45.88	61.17
.2	15.3338	30.67	46.00	61.34
.3	15.3751	30.75	46.13	61.50
.4	15.4165	30.83	46.25	61.66
.5	15.4580	30.92	46.37	61.83
.6	15.4950	30.99	46.49	61.98
.7	15.5411	31.08	46.62	62.16
.8	15.5827	31.17	46.75	62.33
.9	15.6244	31.25	46.87	62.50

DEPTH.				
5	6	7	8	9
72.19	86.63	101.07	115.50	129.94
72.39	86.87	101.35	115.82	130.30
72.59	87.11	101.63	116.14	130.66
72.79	87.35	101.91	116.46	131.02
73.00	87.59	102.19	116.79	131.39
73.20	87.83	102.47	117.11	131.75
73.40	88.07	102.75	117.43	132.11
73.60	88.32	103.04	117.76	132.48
73.80	88.56	103.32	118.08	132.84
74.01	88.81	103.61	118.41	133.21
74.21	89.05	103.89	118.74	133.58
74.41	89.29	104.17	119.06	133.94
74.62	89.54	104.46	119.38	134.31
74.82	89.78	104.75	119.71	134.68
75.03	90.03	105.04	120.04	135.05
75.23	90.28	105.32	120.37	135.41
75.44	90.52	105.61	120.70	135.78
75.64	90.77	105.90	121.02	136.15
75.85	91.01	106.18	121.35	136.52
76.05	91.26	106.47	121.68	136.89
76.26	91.51	106.76	122.01	137.26
76.46	91.75	107.04	122.34	137.63
76.67	92.00	107.34	122.67	138.01
76.88	92.25	107.63	123.00	138.38
77.08	92.50	107.91	123.33	138.74
77.29	92.75	108.21	123.66	139.12
77.48	92.97	108.47	123.96	139.46
77.71	93.25	108.79	124.33	139.87
77.92	93.50	109.08	124.66	140.25
78.12	93.74	109.37	124.99	140.62

Diam. in Inches.	DEPTH.			
	1	2	3	4
75.0	15.6662	31.33	47.00	62.66
.1	15.7080	31.42	47.12	62.83
.2	15.7499	31.50	47.25	63.00
.3	15.7918	31.58	47.38	63.17
.4	15.8337	31.67	47.50	63.34
.5	15.8758	31.75	47.63	63.50
.6	15.9178	31.84	47.75	63.67
.7	15.9600	31.92	47.88	63.84
.8	16.0022	32.00	48.01	64.01
.9	16.0444	32.09	48.13	64.18
76.0	16.0867	32.17	48.26	64.35
.1	16.1291	32.26	48.39	64.52
.2	16.1715	32.34	48.51	64.68
.3	16.2140	32.43	48.64	64.86
.4	16.2565	32.51	48.77	65.02
.5	16.2991	32.60	48.90	65.20
.6	16.3417	32.68	49.03	65.33
.7	16.3844	32.77	49.15	65.54
.8	16.4272	32.85	49.28	65.71
.9	16.4700	32.94	49.41	65.88
77.0	16.5129	33.03	49.54	66.05
.1	16.5558	33.11	49.67	66.22
.2	16.5988	33.20	49.80	66.40
.3	16.6418	33.28	49.93	66.57
.4	16.6849	33.37	50.06	66.74
.5	16.7280	33.46	50.18	66.91
.6	16.7712	33.54	50.31	67.08
.7	16.8145	33.63	50.44	67.26
.8	16.8578	33.72	50.57	67.43
.9	16.9011	33.80	50.70	67.60

DEPTH.				
5	6	7	8	9
78.33	94.00	109.66	125.33	140.99
78.54	94.25	109.96	125.66	141.37
78.75	94.50	110.25	126.00	141.75
78.96	94.75	110.54	126.34	142.13
79.17	95.00	110.84	126.67	142.51
79.38	95.26	111.13	127.01	142.88
79.59	95.51	111.43	127.34	143.26
79.80	95.76	111.72	127.68	143.64
80.01	96.01	112.01	128.02	144.02
80.22	96.26	112.31	128.35	144.40
80.44	96.52	112.61	128.70	144.78
80.65	96.77	112.90	129.03	145.16
80.86	97.03	113.20	129.37	145.54
81.07	97.28	113.50	129.71	145.93
81.28	97.54	113.79	130.05	146.30
81.50	97.80	114.10	130.40	146.70
81.71	98.05	114.39	130.74	147.08
81.92	98.30	114.69	131.07	147.46
82.14	98.56	114.99	131.42	147.84
82.35	98.82	115.29	131.76	148.23
82.57	99.08	115.59	132.10	148.62
82.78	99.34	115.89	132.45	149.00
83.00	99.59	116.19	132.79	149.39
83.21	99.85	116.49	133.14	149.78
83.43	100.11	116.80	133.48	150.17
83.64	100.37	117.10	133.82	150.55
83.86	100.63	117.40	134.17	150.94
84.07	100.88	117.70	134.51	151.33
84.29	101.15	118.01	134.86	151.72
84.51	101.41	118.31	135.21	152.11

Diam. in Inches.	DEPTH.			
	1	2	3	4
78.0	16.9445	33.89	50.83	67.78
.1	16.9880	33.98	50.96	67.95
.2	17.0316	34.06	51.09	68.12
.3	17.0751	34.15	51.23	68.30
.4	17.1188	34.24	51.36	68.48
.5	17.1625	34.32	51.49	68.65
.6	17.2062	34.41	51.62	68.82
.7	17.2500	34.50	51.75	69.00
.8	17.2939	34.59	51.88	69.18
.9	17.3378	34.68	52.01	69.35
79.0	17.3818	34.76	52.15	69.53
.1	17.4258	34.85	52.28	69.70
.2	17.4699	34.94	52.41	69.88
.3	17.5141	35.03	52.54	70.06
.4	17.5583	35.12	52.67	70.23
.5	17.6025	35.20	52.81	70.41
.6	17.6468	35.29	52.94	70.59
.7	17.6912	35.38	53.07	70.76
.8	17.7356	35.47	53.21	70.94
.9	17.7801	35.56	53.34	71.12
80.0	17.8246	35.65	53.47	71.30
.1	17.8692	35.74	53.61	71.48
.2	17.9139	35.83	53.74	71.66
.3	17.9586	35.92	53.87	71.83
.4	18.0033	36.01	54.01	72.01
.5	18.0481	36.10	54.14	72.19
.6	18.0930	36.19	54.28	72.37
.7	18.1379	36.28	54.41	72.55
.8	18.1829	36.37	54.55	72.73
.9	18.2280	36.46	54.68	72.91

DEPTH.				
5	6	7	8	9
84.72	101.66	118.61	135.55	152.50
84.94	101.93	118.92	135.90	152.89
85.16	102.19	119.22	136.25	153.28
85.38	102.45	119.53	136.60	153.68
85.60	102.71	119.83	136.95	154.07
85.81	102.97	120.13	137.30	154.46
86.03	103.24	120.44	137.65	154.85
86.25	103.50	120.75	138.00	155.25
86.47	103.76	121.06	138.35	155.65
86.69	104.03	121.37	138.70	156.04
86.91	104.29	121.67	139.06	156.44
87.13	104.56	121.98	139.41	156.83
87.35	104.82	122.29	139.76	157.23
87.57	105.08	122.60	140.11	157.63
87.79	105.35	122.91	140.46	158.02
88.01	105.61	123.21	140.82	158.42
88.24	105.88	123.52	141.18	158.82
88.46	106.15	123.84	141.53	159.22
88.68	106.41	124.15	141.88	159.62
88.90	106.68	124.46	142.24	160.02
89.12	106.94	124.77	142.59	160.42
89.35	107.21	125.08	142.95	160.82
89.57	107.48	125.40	143.31	161.23
89.79	107.75	125.71	143.66	161.62
90.02	108.02	126.02	144.03	162.03
90.24	108.29	126.34	144.39	162.43
90.47	108.56	126.65	144.74	162.84
90.69	108.83	126.97	145.10	163.24
90.92	109.10	127.28	145.46	163.65
91.14	109.37	127.60	145.82	164.05

Diam. in Inches.	DEPTH.			
	1	2	3	4
81.0	18.2730	36.55	54.82	73.09
.1	18.3182	36.64	54.95	73.27
.2	18.3634	36.73	55.09	73.45
.3	18.4086	36.82	55.22	73.63
.4	18.4540	36.91	55.36	73.82
.5	18.4993	37.00	55.50	74.00
.6	18.5448	37.09	55.64	74.18
.7	18.5902	37.18	55.77	74.36
.8	18.6358	37.27	55.91	74.54
.9	18.6814	37.36	56.04	74.72
82.0	18.7270	37.45	56.18	74.91
.1	18.7727	37.55	56.32	75.09
.2	18.8185	37.64	56.45	75.27
.3	18.8643	37.73	56.59	75.46
.4	18.9102	37.82	56.73	75.64
.5	18.9561	37.91	56.87	75.82
.6	19.0021	38.00	57.01	76.01
.7	19.0481	38.10	57.14	76.19
.8	19.0942	38.19	57.28	76.38
.9	19.1403	38.28	57.42	76.56
83.0	19.1866	38.37	57.56	76.74
.1	19.2328	38.47	57.70	76.93
.2	19.2791	38.56	57.84	77.12
.3	19.3255	38.65	57.97	77.30
.4	19.3719	38.74	58.12	77.49
.5	19.4184	38.84	58.25	77.67
.6	19.4650	38.93	58.40	77.86
.7	19.5115	39.02	58.53	78.04
.8	19.5582	39.12	58.67	78.23
.9	19.6049	39.21	58.82	78.42

DEPTH.				
5	6	7	8	9
91.37	109.64	127.91	146.18	164.46
91.59	109.91	128.23	146.55	164.86
91.82	110.18	128.54	146.91	165.27
92.04	110.45	128.86	147.27	165.67
92.27	110.72	129.18	147.63	166.09
92.50	110.99	129.49	147.99	166.49
92.73	111.27	129.82	148.36	166.91
92.95	111.54	130.13	148.72	167.31
93.18	111.82	130.45	149.09	167.72
93.41	112.09	130.77	149.45	168.13
93.64	112.36	131.09	149.81	168.54
93.87	112.64	131.41	150.18	168.96
94.09	112.91	131.73	150.54	169.36
94.32	113.18	132.05	150.91	169.78
94.55	113.46	132.37	151.28	170.19
94.78	113.74	132.69	151.65	170.60
95.01	114.01	133.01	152.02	171.02
95.24	114.29	133.34	152.39	171.43
95.47	114.56	133.66	152.75	171.85
95.70	114.84	133.98	153.13	172.26
95.93	115.12	134.30	153.49	172.67
96.17	115.40	134.63	153.86	173.10
96.40	115.67	134.95	154.23	173.51
96.63	115.95	135.28	154.60	173.93
96.86	116.23	135.60	154.98	174.35
97.09	116.51	135.93	155.34	174.76
97.33	116.79	136.26	155.72	175.19
97.56	117.07	136.58	156.09	175.60
97.79	117.35	136.91	156.46	176.02
98.03	117.63	137.24	156.84	176.45

Diam. in Inches.	DEPTH.			
	1	2	3	4
84.0	19.6517	39.30	58.96	78.61
.1	19.6985	39.40	59.09	78.79
.2	19.7454	39.49	59.24	78.98
.3	19.7923	39.58	59.38	79.17
.4	19.8393	39.68	59.52	79.36
.5	19.8863	39.77	59.66	79.54
.6	19.9334	39.87	59.80	79.73
.7	19.9806	39.96	59.94	79.92
.8	20.0278	40.06	60.08	80.11
.9	20.0750	40.15	60.23	80.30
85.0	20.1223	40.24	60.37	80.49
.1	20.1697	40.34	60.51	80.68
.2	20.2172	40.43	60.65	80.87
.3	20.2646	40.53	60.79	81.06
.4	20.3122	40.62	60.94	81.25
.5	20.3598	40.72	61.08	81.44
.6	20.4074	40.81	61.22	81.63
.7	20.4551	40.91	61.37	81.82
.8	20.5029	41.01	61.51	82.01
.9	20.5507	41.10	61.65	82.20
86.0	20.5986	41.20	61.79	82.39
.1	20.6465	41.29	61.94	82.58
.2	20.6945	41.39	62.08	82.78
.3	20.7426	41.48	62.23	82.97
.4	20.7907	41.58	62.37	83.16
.5	20.8388	41.68	62.52	83.36
.6	20.8870	41.77	62.66	83.55
.7	20.9353	41.87	62.81	83.74
.8	20.9836	41.97	62.95	83.93
.9	21.0320	42.06	63.10	84.13

DEPTH.				
5	6	7	8	9
98.26	117.91	137.56	157.22	176.87
98.49	118.19	137.89	157.58	177.28
98.73	118.47	138.22	157.96	177.71
98.96	118.75	138.54	158.34	178.13
99.20	119.03	138.87	158.71	178.55
99.43	119.32	139.20	159.09	178.97
99.67	119.60	139.53	159.46	179.40
99.90	119.88	139.86	159.84	179.82
100.14	120.17	140.20	160.22	180.25
100.38	120.45	140.53	160.60	180.68
100.61	120.73	140.85	160.98	181.10
100.86	121.02	141.19	161.36	181.53
101.09	121.30	141.52	161.74	181.95
101.32	121.58	141.85	162.11	182.38
101.56	121.87	142.18	162.50	182.81
101.80	122.16	142.52	162.88	183.24
102.04	122.44	142.85	163.26	183.66
102.28	122.73	143.19	163.64	184.10
102.52	123.02	143.52	164.02	184.53
102.76	123.31	143.86	164.41	184.96
102.99	123.59	144.19	164.78	185.38
103.23	123.88	144.52	165.17	185.82
103.47	124.16	144.86	165.55	186.25
103.71	124.45	145.19	165.94	186.68
103.96	124.75	145.54	166.33	187.12
104.20	125.03	145.87	166.71	187.55
104.44	125.32	146.21	167.10	187.98
104.68	125.61	146.55	167.48	188.42
104.92	125.90	146.88	167.86	188.85
105.16	126.19	147.22	168.26	189.29

Diam. in Inches.	DEPTH.			
	1	2	3	4
87.0	21.0804	42.16	63.24	84.32
.1	21.1289	42.26	63.39	84.52
.2	21.1775	42.35	63.53	84.71
.3	21.2261	42.45	63.68	84.90
.4	21.2747	42.55	63.83	85.10
.5	21.3234	42.65	63.97	85.29
.6	21.3722	42.74	64.12	85.49
.7	21.4210	42.84	64.26	85.68
.8	21.4699	42.94	64.41	85.88
.9	21.5188	43.04	64.56	86.08
88.0	21.5678	43.14	64.70	86.27
.1	21.6169	43.23	64.85	86.47
.2	21.6660	43.33	65.00	86.66
.3	21.7151	43.43	65.15	86.86
.4	21.7643	43.53	65.29	87.06
.5	21.8136	43.63	65.44	87.25
.6	21.8629	43.73	65.59	87.45
.7	21.9123	43.82	65.74	87.65
.8	21.9617	43.92	65.89	87.85
.9	22.0112	44.02	66.03	88.04
89.0	22.0608	44.12	66.18	88.24
.1	22.1104	44.22	66.33	88.44
.2	22.1600	44.32	66.48	88.64
.3	22.2098	44.42	66.63	88.84
.4	22.2595	44.52	66.78	89.04
.5	22.3093	44.62	66.93	89.24
.6	22.3592	44.72	67.08	89.44
.7	22.4092	44.82	67.23	89.64
.8	22.4592	44.92	67.38	89.84
.9	22.5092	45.02	67.53	90.04

DEPTH.				
5	6	7	8	9
105.40	126.48	147.56	168.64	189.72
105.65	126.77	147.90	169.03	190.16
105.89	127.06	148.24	169.42	190.59
106.13	127.36	148.58	169.81	191.03
106.38	127.65	148.93	170.20	191.48
106.62	127.94	149.26	170.58	191.91
106.86	128.23	149.60	170.98	192.35
107.11	128.53	149.95	171.37	192.79
107.35	128.82	150.29	171.76	193.23
107.60	129.11	150.63	172.15	193.67
107.84	129.41	150.98	172.54	194.11
108.09	129.70	151.32	172.94	194.55
108.33	130.00	151.66	173.33	194.99
108.58	130.29	152.01	173.72	195.44
108.82	130.58	152.35	174.11	195.88
109.07	130.88	152.69	174.50	196.32
109.32	131.18	153.04	174.90	196.77
109.56	131.47	153.38	175.30	197.21
109.81	131.77	153.73	175.70	197.66
110.06	132.07	154.08	176.09	198.10
110.31	132.37	154.43	176.49	198.55
110.55	132.66	154.77	176.88	198.99
110.80	132.96	155.12	177.28	199.44
111.05	133.26	155.47	177.68	199.89
111.30	133.55	155.81	178.07	200.33
111.55	133.85	156.16	178.47	200.78
111.80	134.15	156.51	178.87	201.23
112.05	134.45	156.86	179.27	201.68
112.30	134.75	157.21	179.67	202.13
112.55	135.05	157.56	180.07	202.58

Diam. in Inches.	DEPTH.			
	1	2	3	4
90.0	22.5593	45.12	67.68	90.24
.1	22.6095	45.22	67.83	90.44
.2	22.6597	45.32	67.98	90.64
.3	22.7100	45.42	68.13	90.84
.4	22.7603	45.52	68.28	91.04
.5	22.8107	45.62	68.43	91.24
.6	22.8611	45.72	68.58	91.44
.7	22.9116	45.82	68.73	91.64
.8	22.9621	45.92	68.89	91.85
.9	23.0128	46.03	69.04	92.05
91.0	23.0634	46.13	69.19	92.25
.1	23.1141	46.23	69.34	92.46
.2	23.1649	46.33	69.50	92.66
.3	23.2157	46.43	69.65	92.86
.4	23.2666	46.53	69.80	93.06
.5	23.3176	46.63	69.95	93.27
.6	23.3685	46.74	70.10	93.47
.7	23.4196	46.84	70.26	93.68
.8	23.4707	46.94	70.41	93.88
.9	23.5219	47.04	70.57	94.09
92.0	23.5731	47.15	70.72	94.29
.1	23.6244	47.25	70.87	94.50
.2	23.6757	47.35	71.03	94.70
.3	23.7271	47.45	71.18	94.91
.4	23.7785	47.56	71.33	95.11
.5	23.8300	47.66	71.49	95.32
.6	23.8816	47.76	71.64	95.52
.7	23.9332	47.87	71.80	95.73
.8	23.9848	47.97	71.96	95.94
.9	24.0366	48.07	72.11	96.14

DEPTH.				
5	6	7	8	9
112.80	135.35	157.91	180.47	203.03
113.05	135.65	158.26	180.87	203.48
113.30	135.96	158.62	181.28	203.94
113.55	136.26	158.97	181.68	204.39
113.80	136.56	159.32	182.08	204.84
114.05	136.86	159.67	182.48	205.29
114.31	137.17	160.03	182.89	205.75
114.56	137.47	160.38	183.29	206.20
114.81	137.77	160.73	183.70	206.66
115.07	138.08	161.09	184.10	207.12
115.32	138.38	161.44	184.50	207.57
115.57	138.68	161.80	184.91	208.03
115.83	138.99	162.16	185.32	208.49
116.08	139.30	162.51	185.73	208.94
116.33	139.60	162.86	186.13	209.39
116.59	143.90	163.22	186.54	209.85
116.84	140.21	163.58	186.94	210.31
117.10	140.51	163.93	187.35	210.77
117.36	140.83	164.30	187.77	211.24
117.61	141.13	164.65	188.18	211.70
117.87	141.44	165.01	188.58	212.16
118.12	141.74	165.37	188.99	212.62
118.38	142.05	165.73	189.40	213.08
118.64	142.36	166.09	189.82	213.54
118.89	142.67	166.45	190.22	214.00
119.15	142.98	166.81	190.64	214.47
119.41	143.29	167.17	191.05	214.93
119.67	143.60	167.53	191.46	215.40
119.93	143.91	167.90	191.88	215.87
120.18	144.22	168.25	192.29	216.32

Diam. in Inches.	DEPTH.			
	1	2	3	4
93.0	24.0883	48.18	72.26	96.35
.1	24.1402	48.28	72.42	96.56
.2	24.1920	48.38	72.58	96.77
.3	24.2440	48.49	72.73	96.98
.4	24.2960	48.59	72.89	97.18
.5	24.3480	48.70	73.04	97.39
.6	24.4001	48.80	73.20	97.60
.7	24.4523	48.90	73.36	97.81
.8	24.5045	49.01	73.51	98.02
.9	24.5568	49.11	73.67	98.23
94.0	24.6091	49.22	73.83	98.44
.1	24.6615	49.32	73.98	98.64
.2	24.7140	49.43	74.14	98.86
.3	24.7665	49.53	74.30	99.06
.4	24.8190	49.64	74.46	99.28
.5	24.8716	49.74	74.61	99.48
.6	24.9243	49.85	74.77	99.70
.7	24.9770	49.95	74.93	99.91
.8	25.0298	50.06	75.09	100.12
.9	25.0826	50.16	75.25	100.33
95.0	25 1355	50.27	75.41	100.54
.1	25.1885	50.38	75.56	100.75
.2	25.2415	50.48	75.72	100.96
.3	25.2945	50.59	75.88	101.18
.4	25.3476	50.69	76.04	101.39
.5	25.4008	50.80	76.20	101.60
.6	25.4540	50.91	76.36	101.82
.7	25.5073	51.01	76.52	102.03
.8	25.5606	51.12	76.68	102.24
.9	25.6140	51.23	76.84	102.46

DEPTH.				
5	6	7	8	9
120.44	144.53	168.61	192.70	216.79
120.70	144.84	168.98	193.12	217.26
120.96	145.15	169.34	193.54	217.73
121.22	145.46	169.70	193.95	218.20
121.48	145.78	170.07	194.37	218.66
121.74	146.09	170.43	194.78	219.13
122.00	146.40	170.80	195.20	219.60
122.26	146.71	171.16	195.62	220.07
122.52	147.02	171.52	196.03	220.54
122.79	147.34	171.89	196.46	221.01
123.05	147.65	172.26	196.87	221.48
123.31	147.97	172.62	197.29	221.95
123.57	148.28	173.00	197.71	222.43
123.83	148.60	173.36	198.13	222.89
124.10	148.91	173.73	198.55	223.37
124.36	149.23	174.09	198.97	223.84
124.62	149.54	174.46	199.39	224.32
124.89	149.86	174.83	199.82	224.79
125.15	150.18	175.21	200.24	225.27
125.41	150.49	175.57	200.66	225.74
125.68	150.81	175.94	201.08	226.22
125.94	151.13	176.31	201.50	226.69
126.21	151.45	176.68	201.93	227.17
126.47	151.76	177.05	202.35	227.65
126.74	152.08	177.42	202.78	228.12
127.01	152.41	177.80	203.21	228.61
127.27	152.72	178.17	203.63	229.09
127.54	153.04	178.54	204.06	229.56
127.80	153.36	178.92	204.48	230.04
128.07	153.68	179.29	204.91	230.53

Diam. in Inches.	DEPTH.			
	1	2	3	4
96.0	25.6675	51.33	77.00	102.67
.1	25.7210	51.44	77.16	102.88
.2	25.7745	51.55	77.32	103.10
.3	25.8282	51.66	77.48	103.31
.4	25.8818	51.76	77.65	103.53
.5	25.9355	51.87	77.81	103.74
.6	25.9893	51.98	77.97	103.96
.7	26.0432	52.09	78.13	104.17
.8	26.0971	52.19	78.29	104.39
.9	26.1510	52.30	78.45	104.60
97.0	26.2050	52.41	78.62	104.82
.1	26.2591	52.52	78.78	105.04
.2	26.3132	52.63	78.94	105.25
.3	26.3673	52.73	79.10	105.47
.4	26.4216	52.84	79.26	105.68
.5	26.4759	52.95	79.43	105.90
.6	26.5302	53.06	79.59	106.12
.7	26.5846	53.17	79.75	106.34
.8	26.6390	53.28	79.92	106.56
.9	26.6935	53.39	80.08	106.77
98.0	26.7481	53.50	80.24	106.99
.1	26.8027	53.61	80.41	107.21
.2	26.8574	53.71	80.57	107.43
.3	26.9121	53.82	80.74	107.65
.4	26.9669	53.93	80.90	107.87
.5	27.0217	54.04	81.07	108.09
.6	27.0766	54.15	81.23	108.30
.7	27.1316	54.26	81.39	108.52
.8	27.1866	54.37	81.56	108.74
.9	27.2416	54.48	81.72	108.96

DEPTH.				
5	6	7	8	9
128.34	154.00	179.67	205.34	231.00
128.61	154.33	180.05	205.77	231.49
128.87	154.64	180.42	206.19	231.97
129.14	154.97	180.80	206.62	232.45
129.41	155.29	181.17	207.06	232.94
129.68	155.61	181.53	207.48	233.42
129.95	155.93	181.92	207.91	233.90
130.22	156.26	182.30	208.34	234.39
130.49	156.58	182.68	208.78	234.87
130.76	156.91	183.06	209.21	235.36
131.03	157.23	183.44	209.64	235.85
131.30	157.55	183.81	210.07	236.33
131.57	157.88	184.19	210.50	236.82
131.84	158.20	184.57	210.94	237.30
132.11	158.53	184.95	211.37	237.79
132.38	158.86	185.33	211.81	238.28
132.65	159.18	185.71	212.24	238.77
132.92	159.50	186.09	212.67	239.26
133.20	159.83	186.47	213.11	239.75
133.46	160.16	186.85	213.54	240.24
133.74	160.49	187.24	213.98	240.73
134.02	160.82	187.62	214.42	241.23
134.29	161.14	188.00	214.86	241.71
134.56	161.47	188.38	215.30	242.21
134.84	161.80	188.77	215.74	242.70
135.11	162.13	189.15	216.18	243.20
135.38	162.46	189.53	216.61	243.68
135.66	162.79	189.92	217.05	244.18
135.93	163.12	190.30	217.49	244.67
136.21	163.45	190.69	217.93	245.17

Diam. in Inches.	DEPTH			
	1	2	3	4
99.0	27.2968	54.59	81.89	109.19
.1	27.3519	54.70	82.06	109.41
.2	27.4072	54.81	82.22	109.63
.3	27.4625	54.92	82.39	109.85
.4	27.5178	55.04	82.55	110.07
.5	27.5732	55.15	82.72	110.29
.6	27.6286	55.26	82.88	110.51
.7	27.6841	55.37	83.05	110.74
.8	27.7397	55.48	83.22	110.96
.9	27.7953	55.59	83.39	111.18
100.0	27.8510	55.70	83.55	111.40
.1	27.9067	55.81	83.72	111.63
.2	27.9625	55.92	83.89	111.85
.3	28.0184	56.04	84.05	112.07
.4	28.0743	56.15	84.22	112.30
.5	28.1302	56.26	84.39	112.52
.6	28.1862	56.37	84.56	112.74
.7	28.2423	56.48	84.73	112.97
.8	28.2984	56.60	84.89	113.19
.9	28.3546	56.71	85.06	113.42
101.0	28.4108	56.82	85.23	113.64
.1	28.4671	56.93	85.40	113.87
.2	28.5234	57.05	85.57	114.09
.3	28.5798	57.16	85.74	114.32
.4	28.6363	57.27	85.91	114.54
.5	28.6928	57.39	86.08	114.77
.6	28.7494	57.50	86.25	115.00
.7	28.8060	57.61	86.42	115.22
.8	28.8627	57.73	86.59	115.45
.9	28.9194	57.84	86.76	115.68

DEPTH.				
5	6	7	8	9
136.49	163.78	191.08	218.38	245.67
136.76	164.11	191.46	218.82	246.17
137.04	164.44	191.85	219.26	246.66
137.31	164.77	192.23	219.70	247.16
137.59	165.11	192.63	220.14	247.66
137.87	165.44	193.01	220.58	248.16
138.14	165.77	193.40	221.02	248.65
138.42	166.10	193.79	221.47	249.16
138.70	166.44	194.18	221.92	249.66
138.98	166.77	194.57	222.36	250.16
139.26	167.11	194.96	222.81	250.66
139.54	167.44	195.35	223.26	251.16
139.81	167.77	195.73	223.70	251.66
140.09	168.11	196.13	224.14	252.16
140.37	168.44	196.52	224.59	252.67
140.65	168.78	196.91	225.04	253.17
140.93	169.12	197.30	225.49	253.67
141.21	169.45	197.69	225.94	254.18
141.49	169.79	198.09	226.38	254.68
141.77	170.12	198.48	226.83	255.19
142.06	170.47	198.88	227.29	255.70
142.34	170.80	199.27	227.74	256.20
142.62	171.14	199.66	228.18	256.71
142.90	171.48	200.06	228.64	257.22
143.18	171.82	200.45	229.09	257.72
143.47	172.16	200.85	229.54	258.24
143.75	172.49	201.24	229.99	258.74
144.03	172.84	201.64	230.45	259.25
144.32	173.18	202.04	230.90	259.77
144.60	173.51	202.43	231.35	260.27

Diam. in Inches.	DEPTH.			
	1	2	3	4
102.0	28.9762	57.95	86.93	115.90
.1	29.0339	58.07	87.10	116.13
.2	29.0899	58.18	87.27	116.36
.3	29.1469	58.29	87.44	116.59
.4	29.2039	58.41	87.61	116.82
.5	29.2610	58.52	87.78	117.04
.6	29.3181	58.64	87.95	117.27
.7	29.3753	58.75	88.13	117.50
.8	29.4325	58.86	88.30	117.73
.9	29.4898	58.98	88.47	117.96
103.0	29.5471	59.09	88.64	118.19
.1	29.6045	59.21	88.81	118.42
.2	29.6620	59.32	88.99	118.65
.3	29.7195	59.44	89.16	118.88
.4	29.7771	59.55	89.33	119.11
.5	29.8347	59.67	89.51	119.34
.6	29.8924	59.78	89.68	119.57
.7	29.9501	59.90	89.85	119.80
.8	30.0079	60.02	90.02	120.03
.9	30.0657	60.13	90.20	120.26
104.0	30.1236	60.25	90.37	120.49
.1	30.1816	60.36	90.54	120.72
.2	30.2396	60.48	90.72	120.96
.3	30.2977	60.60	90.89	121.19
.4	30.3558	60.71	91.07	121.42
.5	30.4140	60.81	91.24	121.66
.6	30.4722	60.94	91.42	121.89
.7	30.5305	61.06	91.59	122.12
.8	30.5889	61.18	91.77	122.36
.9	30.6473	61.29	91.94	122.59

DEPTH.				
5	6	7	8	9
144.88	173.86	202.83	231.81	260.78
145.17	174.20	203.23	232.26	261.30
145.45	174.54	203.63	232.72	261.81
145.74	174.88	204.03	233.18	262.32
146.02	175.22	204.43	233.63	262.84
146.31	175.57	204.83	234.09	263.35
146.59	175.91	205.23	234.54	263.86
146.88	176.25	205.63	235.00	264.38
147.16	176.59	206.02	235.46	264.89
147.45	176.94	206.43	235.92	265.41
147.74	177.28	206.83	236.38	265.92
148.02	177.62	207.23	236.83	266.44
148.31	177.97	207.63	237.30	266.96
148.60	178.31	208.03	237.75	267.47
148.89	178.66	208.44	238.22	267.99
149.18	179.01	208.85	238.68	268.52
149.46	179.35	209.24	239.14	269.03
149.76	179.70	209.65	239.60	269.55
150.04	180.05	210.06	240.06	270.07
150.33	180.40	210.46	240.53	270.59
150.62	180.74	210.86	240.98	271.11
150.91	181.09	211.27	241.45	271.63
151.20	181.43	211.67	241.91	272.15
151.49	181.79	212.09	242.38	272.68
151.78	182.14	212.49	242.85	273.20
152.07	182.48	212.90	243.31	273.73
152.36	182.83	213.30	243.78	274.25
152.65	183.18	213.71	244.24	274.77
152.95	183.53	214.12	244.71	275.30
153.24	183.88	214.53	245.18	275.82

Diam. in Inches.	DEPTH.			
	1	2	3	4
105.0	30.7057	61.41	92.12	122.82
.1	30.7642	61.53	92.29	123.06
.2	30.8228	61.65	92.47	123.29
.3	30.8814	61.76	92.64	123.52
.4	30.9401	61.88	92.82	123.76
.5	30.9989	62.00	93.00	124.00
.6	31.0577	62.12	93.17	124.23
.7	31.1165	62.23	93.35	124.46
.8	31.1754	62.35	93.53	124.70
.9	31.2344	62.47	93.70	124.94
106.0	31.2934	62.59	93.88	125.17
.1	31.3525	62.70	94.06	125.41
.2	31.4116	62.82	94.23	125.64
.3	31.4708	62.94	94.41	125.88
.4	31.5300	63.06	94.59	126.12
.5	31.5893	63.18	94.77	126.36
.6	31.6487	63.30	94.95	126.60
.7	31.7081	63.42	95.12	126.83
.8	31.7675	63.53	95.30	127.07
.9	31.8270	63.65	95.48	127.30
107.0	31.8866	63.77	95.66	127.54
.1	31.9462	63.89	95.84	127.78
.2	32.0059	64.01	96.02	128.02
.3	32.0657	64.13	96.20	128.26
.4	32.1255	64.25	96.38	128.50
.5	32.1853	64.37	96.56	128.74
.6	32.2452	64.49	96.74	128.98
.7	32.3052	64.61	96.92	129.22
.8	32.3652	64.73	97.10	129.46
.9	32.4252	64.85	97.28	129.70

DEPTH.				
5	6	7	8	9
153.53	184.24	214.94	245.65	276.35
153.82	184.58	215.35	246.11	276.88
154.12	184.94	215.76	246.58	277.41
154.41	185.29	216.17	247.05	277.93
154.70	185.64	216.58	247.52	278.46
155.00	185.99	216.99	247.99	278.99
155.29	186.35	217.41	248.46	279.52
155.58	186.70	217.81	248.93	280.04
155.88	187.05	218.23	249.40	280.58
156.17	187.40	218.64	249.87	281.11
156.47	187.76	219.05	250.34	281.64
156.76	188.11	219.46	250.82	282.17
157.06	188.47	219.88	251.29	282.70
157.36	188.83	220.30	251.77	283.24
157.65	189.18	220.71	252.24	283.77
157.95	189.53	221.12	252.71	284.30
158.25	189.89	221.54	253.19	284.84
158.54	190.25	221.96	253.66	285.37
158.84	190.60	222.37	254.14	285.90
159.14	190.96	222.79	254.62	286.44
159.43	191.32	223.20	255.09	286.97
159.73	191.68	223.62	255.57	287.51
160.03	192.04	224.04	256.05	288.05
160.33	192.40	224.46	256.53	288.59
160.63	192.75	224.88	257.00	289.12
160.93	193.11	225.30	257.48	289.67
161.23	193.47	225.72	257.96	290.21
161.53	193.83	226.14	258.44	290.75
161.83	194.19	226.56	258.92	291.29
162.13	194.55	226.98	259.40	291.82

Diam. in Inches.	DEPTH.			
	1	2	3	4
108.0	32.4854	64.97	97.46	129.94
.1	32.5456	65.09	97.64	130.18
.2	32.6058	65.21	97.82	130.42
.3	32.6661	65.33	98.00	130.66
.4	32.7265	65.45	98.18	130.90
.5	32.7869	65.57	98.36	131.15
.6	32.8474	65.69	98.54	131.39
.7	32.9079	65.82	98.72	131.63
.8	32.9685	65.94	98.90	131.87
.9	33.0291	66.06	99.09	132.12
109.0	33.0898	66.18	99.27	132.36
.1	33.1505	66.30	99.45	132.60
.2	33.2113	66.42	99.63	132.84
.3	33.2722	66.54	99.82	133.09
.4	33.3331	166.67	100.00	133.33
.5	33.3940	66.79	100.18	133.58
.6	33.4551	66.91	100.37	133.82
.7	33.5161	67.03	100.55	134.06
.8	33.5773	67.16	100.73	134.31
.9	33.6385	67.28	100.92	134.56
110.0	33.6997	67.40	101.10	134.80
.1	33.7610	67.52	101.28	135.04
.2	33.8224	67.64	101.47	135.29
.3	33.8838	67.77	101.65	135.54
.4	33.9452	67.89	101.84	135.78
.5	34.0068	68.01	102.02	136.03
.6	34.0683	68.14	102.20	136.27
.7	34.1300	68.26	102.39	136.52
.8	34.1917	68.38	102.58	136.77
.9	34.2534	68.51	102.76	137.01

DEPTH.				
5	6	7	8	9
162.43	194.91	227.40	259.88	292.37
162.73	195.27	227.82	260.36	292.91
163.03	195.63	228.24	260.85	293.45
163.33	196.00	228.66	261.33	293.99
163.63	196.36	229.08	261.81	294.53
163.94	196.72	229.51	262.30	295.08
164.24	197.08	229.93	262.78	295.62
164.54	197.45	230.36	263.26	296.17
164.84	197.80	230.78	263.74	296.71
165.15	198.17	231.20	264.23	297.26
165.45	198.54	231.63	264.72	297.81
165.75	198.90	232.05	265.20	298.35
166.06	199.27	232.48	265.69	298.90
166.36	199.63	232.90	266.18	299.45
166.67	200.00	233.33	266.66	300.00
166.97	200.36	233.76	267.15	300.55
167.28	200.73	234.19	267.64	301.10
167.58	201.10	234.61	268.13	301.64
167.89	201.47	235.05	268.62	302.20
168.20	201.83	235.47	269.11	302.75
168.50	202.20	235.90	269.60	303.30
168.81	202.57	236.33	270.09	303.85
169.11	202.93	236.75	270.58	304.40
169.42	203.30	237.19	271.07	304.96
169.73	203.67	237.62	271.56	305.51
170.04	204.04	238.05	272.06	306.06
170.34	204.41	238.48	272.54	306.61
170.65	204.78	238.91	273.04	307.17
170.96	205.15	239.34	273.54	307.73
171.07	205.52	239.77	274.02	308.28

Diam. in Inches.	DEPTH.			
	1	2	3	4
111.0	34.3152	68.63	102.95	137.26
.1	34.3771	68.75	103.13	137.51
.2	34.4390	68.88	103.32	137.76
.3	34.5010	69.00	103.50	138.00
.4	34.5630	69.13	103.69	138.25
.5	34.6251	69.25	103.88	138.50
.6	34.6872	69.37	104.06	138.75
.7	34.7494	69.50	194.25	139.00
.8	34.8116	69.62	104.43	139.24
.9	34.8739	69.75	104.62	139.50
112.0	34.9363	69.87	104.81	139.74
.1	34.9987	70.00	105.00	140.00
.2	35.0612	70.12	105.18	140.24
.3	35.1237	70.25	105.37	140.50
.4	35.1863	70.37	105.56	140.74
.5	35.2489	70.50	105.75	141.00
.6	35.3116	70.62	105.93	141.24
.7	35.3744	70.75	106.12	141.50
.8	35.4372	70.87	106.31	141.75
.9	35.5000	71.00	106.50	142.00
113.0	35.5629	71.13	106.69	142.25
.1	35.6259	71.25	106.88	142.50
.2	35.6889	71.38	107.07	142.76
.3	35.7520	71.50	107.26	143.01
.4	35.8152	71.63	107.45	143.26
.5	35.8784	71.76	107.64	143.51
.6	35.9416	71.88	107.82	143.76
.7	36.0049	72.01	108.02	144.02
.8	36.0683	72.14	108.21	144.27
.9	36.1317	72.26	108.40	144.53

DEPTH.				
5	6	7	8	9
171.58	205.89	240.21	274.52	308.84
171.89	206.26	240.64	275.02	309.39
172.20	206.63	241.07	275.51	309.95
172.51	207.01	241.51	276.01	310.51
172.82	207.38	241.94	276.50	311.07
173.13	207.75	242.38	277.00	311.63
173.44	208.12	242.81	277.50	312.18
173.75	208.49	243.24	277.99	312.74
174.06	208.87	243.68	278.49	313.30
174.37	209.24	244.12	278.99	313.87
174.68	209.62	244.55	279.49	314.42
175.00	209.99	244.99	279.99	314.99
175.31	210.37	245.43	280.49	315.55
175.62	210.74	245.87	280.99	316.12
175.93	211.12	246.30	281.49	316.67
176.25	211.49	246.84	281.99	317.24
176.56	211.87	247.18	282.49	317.80
176.87	212.24	247.62	282.99	318.37
177.19	212.62	248.06	283.50	318.93
177.50	213.00	248.50	284.00	319.50
177.82	213.38	248.94	284.50	320.07
178.13	213.76	249.38	285.01	320.63
178.45	214.13	249.82	285.51	321.20
178.76	214.51	250.26	286.02	321.77
179.08	214.89	250.71	286.52	322.34
179.39	215.27	251.15	287.02	322.90
179.71	215.65	251.59	287.53	323.47
180.03	216.03	252.04	288.04	324.05
180.34	216.41	252.48	288.54	324.61
180.66	216.79	252.92	289.06	325.19

Diam. in Inches.	DEPTH.			
	1	2	3	4
114.0	36.1952	72.39	108.59	144.78
.1	36.2587	72.52	108.78	145.04
.2	36.3223	72.64	108.97	145.29
.3	36.3859	72.77	109.16	145.54
.4	36.4496	72.90	109.35	145.80
.5	36.5134	73.03	109.54	146.05
.6	36.5772	73.15	109.73	146.31
.7	36.6410	73.28	109.92	146.56
.8	36.7049	73.41	110.12	146.82
.9	36.7689	73.54	110.31	147.08
115.0	36.8329	73.67	110.50	147.33
.1	36.8970	73.79	110.69	147.59
.2	36.9612	73.92	110.88	147.84
.3	37.0254	74.05	111.08	148.10
.4	37.0896	74.18	111.27	148.36
.5	37.1539	74.31	111.46	148.62
.6	37.2183	74.44	111.66	148.87
.7	37.2827	74.57	111.85	149.13
.8	37.3472	74.69	112.04	149.39
.9	37.4117	74.82	112.24	149.65
116.0	37.4763	74.95	112.43	149.91
.1	37.5409	75.08	112.62	150.16
.2	37.6056	75.21	112.82	150.42
.3	37.6704	75.34	113.01	150.68
.4	37.7352	75.47	113.21	150.94
.5	37.8001	75.60	113.40	151.20
.6	37.8650	75.73	113.60	151.46
.7	37.9300	75.86	113.79	151.72
.8	37.9950	75.99	113.99	151.98
.9	38.0601	76.12	114.18	152.24

DEPTH.				
5	6	7	8	9
180.98	217.17	253.36	289.56	325.76
181.30	217.55	253.81	290.07	326.33
181.61	217.93	254.25	290.58	326.90
181.93	218.32	254.70	291.09	327.47
182.25	218.69	255.14	291.59	328.04
182.57	219.08	255.59	292.10	328.62
182.89	219.46	256.04	292.62	329.19
183.21	219.85	256.49	293.13	329.77
183.53	220.23	256.94	293.64	330.35
183.85	220.61	257.38	294.15	330.92
184.17	221.00	257.83	294.66	331.50
184.49	221.38	258.28	295.18	332.07
184.81	221.77	258.73	295.69	332.65
185.13	222.15	259.18	296.20	333.23
185.45	222.53	259.62	296.71	333.80
185.78	222.92	260.08	297.23	334.39
186.09	223.31	260.53	297.74	334.96
186.42	223.70	260.98	298.27	335.55
186.74	224.08	261.43	298.78	336.12
187.06	224.47	261.88	299.30	336.71
187.38	224.86	262.33	299.81	337.29
187.71	225.25	262.79	300.33	337.87
188.03	225.63	263.24	300.84	338.45
188.35	226.02	263.69	301.36	339.03
188.68	226.41	264.15	301.88	339.62
189.00	226.80	264.60	302.40	340.20
189.33	227.19	265.06	302.92	340.79
189.65	227.58	265.51	303.44	341.37
189.98	227.97	265.97	303.96	341.96
190.30	228.36	266.42	304.48	342.54

Diam. in Inches.	DEPTH.			
	1	2	3	4
117.0	38.1252	76.25	114.38	152.50
.1	38.1904	76.38	114.57	152.76
.2	38.2557	76.51	114.77	153.02
.3	38.3210	76.64	114.96	153.28
.4	38.3864	76.77	115.16	153.54
.5	38.4518	76.90	115.36	153.81
.6	38.5173	77.03	115.55	154.07
.7	38.5828	77.17	115.75	154.33
.8	38.6484	77.30	115.95	154.59
.9	38.7140	77.43	116.14	154.86
118.0	38.7797	77.56	116.34	155.12
.1	38.8455	77.69	116.54	155.38
.2	38.9113	77.82	116.73	155.64
.3	38.9772	77.95	116.93	155.91
.4	39.0431	78.09	117.13	156.17
.5	39.1091	78.22	117.33	156.44
.6	39.1751	78.35	117.53	156.70
.7	39.2412	78.48	117.72	156.96
.8	39.3073	78.61	117.92	157.23
.9	39.3735	78.75	118.12	157.49
119.0	39.4398	78.88	118.32	157.76
.1	39.5061	79.01	118.52	158.02
.2	39.5725	79.14	118.72	158.29
.3	39.6389	79.28	118.92	158.56
.4	39.7054	79.41	119.12	158.82
.5	39.7719	79.54	119.32	159.09
.6	39.8385	79.68	119.51	159.35
.7	39.9052	79.81	119.72	159.62
.8	39.9719	79.94	119.92	159.89
.9	40.0386	80.08	120.11	160.15

DEPTH.				
5	6	7	8	9
190.63	228.75	266.88	305.00	343.13
190.95	229.14	267.33	305.52	343.71
191.28	229.54	267.79	306.05	344.30
191.61	229.93	268.25	306.57	344.89
191.93	230.32	268.70	307.09	345.47
192.26	230.71	269.16	307.62	346.07
192.59	231.10	269.62	308.14	346.65
192.92	231.50	270.08	308.66	347.25
193.24	231.89	270.54	309.18	347.83
193.57	232.28	271.00	309.71	348.43
193.90	232.68	271.46	310.24	349.02
194.23	233.07	271.92	310.76	349.61
194.56	233.47	272.38	311.29	350.20
194.89	233.86	272.84	311.82	350.79
195.22	234.26	273.30	312.34	351.39
195.55	234.65	273.76	312.87	351.98
195.88	235.06	274.23	313.40	352.58
196.21	235.45	274.69	313.93	353.17
196.54	235.84	275.15	314.46	353.76
196.87	236.24	275.61	314.98	354.36
197.20	236.64	276.08	315.52	354.96
197.53	237.04	276.54	316.05	355.55
197.86	237.43	277.01	316.58	356.15
198.20	237.83	277.47	317.11	356.75
198.53	238.23	277.94	317.64	357.35
198.86	238.63	278.40	318.18	357.95
199.19	239.03	278.87	318.70	358.54
199.53	239.43	279.34	319.24	359.15
199.86	239.83	279.80	319.78	359.75
200.19	240.23	280.27	320.30	360.34

Diam. in Inches.	DEPTH			
	1	2	3	4
120.0	40.1054	80.21	120.32	160.42
.1	40.1723	80.34	120.52	160.69
.2	40.2392	80.48	120.72	160.96
.3	40.3062	80.61	120.92	161.22
.4	40.3733	80.75	121.12	161.49
.5	40.4403	80.88	121.32	161.76
.6	40.5075	81.01	121.52	162.03
.7	40.5747	81.15	121.73	162.30
.8	40.6420	81.28	121.93	162.57
.9	40.7093	81.42	122.13	162.84
121.0	40.7766	81.55	122.33	163.10
.1	40.8441	81.69	122.53	163.38
.2	40.9116	81.82	122.74	163.64
.3	40.9791	81.96	122.94	163.92
.4	41.0467	82.09	123.14	164.19
.5	41.1143	82.23	123.34	164.46
.6	41.1820	82.36	123.55	164.73
.7	41.2498	82.50	123.75	165.00
.8	41.3176	82.63	123.95	165.27
.9	41.3855	82.77	124.16	165.54
122.0	41.4534	82.91	124.36	165.81
.1	41.5214	83.04	124.56	166.08
.2	41.5895	83.18	124.77	166.36
.3	41.6575	83.31	124.97	166.63
.4	41.7257	83.45	125.18	166.90
.5	41.7939	83.59	125.38	167.18
.6	41.8622	83.72	125.59	167.45
.7	41.9305	83.86	125.79	167.72
.8	41.9989	84.00	126.00	168.00
.9	42.0673	84.13	126.20	168.27

DEPTH.				
5	6	7	8	9
200.53	240.63	280.74	320.84	360.95
200.86	241.03	281.20	321.38	361.55
201.20	241.43	281.67	321.91	362.15
201.53	241.84	282.14	322.45	362.75
201.87	242.24	282.61	322.98	363.36
202.20	242.64	283.08	323.52	363.96
202.54	243.04	283.55	324.06	364.56
202.88	243.45	284.02	324.60	365.18
203.21	243.85	284.49	325.14	365.78
203.55	244.25	284.96	325.67	366.38
203.88	244.66	285.43	326.21	366.98
204.22	245.06	285.91	326.75	367.60
204.56	245.47	286.38	327.30	368.21
204.90	245.87	286.85	327.83	368.81
205.24	246.28	287.33	328.38	369.42
205.57	246.68	287.80	328.91	370.03
205.91	247.09	288.27	329.46	370.64
206.25	247.50	288.75	330.00	371.25
206.59	247.90	289.22	330.54	371.85
206.93	248.31	289.70	331.08	372.47
207.27	248.72	290.17	331.62	373.08
207.61	249.13	290.65	332.17	373.69
207.95	249.53	291.12	332.71	374.30
208.29	249.94	291.60	333.26	374.91
208.63	250.36	292.08	333.81	375.53
208.97	250.76	292.56	334.45	376.15
209.31	251.17	293.03	334.90	376.76
209.65	251.58	293.51	335.44	377.37
210.00	251.99	293.99	335.99	377.99
210.34	252.40	294.47	336.54	378.60

Diam. in Inches.	DEPTH.			
	1	2	3	4
123.0	42.1358	84.27	126.41	168.55
.1	42.2043	84.41	126.61	168.82
.2	42.2729	84.55	126.82	169.09
.3	42.3416	84.68	127.02	169.36
.4	42.4103	84.82	127.23	169.64
.5	42.4790	84.96	127.44	169.92
.6	42.5479	85.10	127.64	170.19
.7	42.6167	85.23	127.85	170.47
.8	42.6857	85.37	128.06	170.74
.9	42.7547	85.51	128.27	171.02
124.0	42.8237	85.65	128.47	171.29
.1	42.8928	85.79	128.68	171.57
.2	42.9619	85.92	128.89	171.85
.3	43.0312	86.06	129.09	172.12
.4	43.1004	86.20	129.30	172.40
.5	43.1697	86.34	129.51	172.68
.6	43.2391	86.48	129.72	172.96
.7	43.3086	86.62	129.92	173.23
.8	43.3780	86.76	130.13	173.51
.9	43.4476	86.89	130.34	173.79
125.0	43.5172	87.03	130.55	174.07
.1	43.5868	87.17	130.76	174.35
.2	43.6566	87.31	130.97	174.62
.3	43.7263	87.45	131.18	174.90
.4	43.7961	87.59	131.39	175.18
.5	43.8660	87.73	131.60	175.46
.6	43.9360	87.87	131.81	175.74
.7	44.0059	88.01	132.02	176.02
.8	44.0760	88.15	132.23	176.30
.9	44.1461	88.29	132.44	176.58

DEPTH.				
5	6	7	8	9
210.68	252.82	294.95	337.09	379.22
211.02	253.22	295.43	337.63	379.84
211.37	253.64	295.91	338.18	380.46
211.71	254.05	296.39	338.73	381.07
212.05	254.46	296.87	339.28	381.69
212.40	254.87	297.35	339.83	382.31
212.74	255.29	297.84	340.38	382.93
213.09	255.70	298.32	340.94	383.55
213.43	256.12	298.80	341.49	384.17
213.78	256.53	299.29	342.04	384.80
214.12	256.94	299.76	342.58	385.41
214.47	257.36	300.25	343.14	386.04
214.81	257.77	300.73	343.70	386.66
215.16	258.19	301.22	344.25	387.28
215.50	258.60	301.70	344.80	387.90
215.85	259.02	302.19	345.36	388.53
216.20	259.43	302.67	345.91	389.15
216.54	259.85	303.16	346.46	389.77
216.89	260.27	303.65	347.02	390.40
217.24	260.68	304.13	347.58	391.02
217.59	261.10	304.62	348.14	391.65
217.94	261.52	305.11	348.70	392.28
218.28	261.94	305.59	349.25	392.90
218.63	262.36	306.08	349.81	393.53
218.98	262.78	306.57	350.37	394.16
219.33	263.20	307.06	350.93	394.79
219.68	263.62	307.55	351.49	395.42
220.03	264.04	308.04	352.05	396.05
220.38	264.46	308.53	352.61	396.68
220.73	264.88	309.02	353.17	397.31

Diam. in Inches.	DEPTH.			
	1	2	3	4
126.0	44.2162	88.43	132.65	176.86
.1	44.2865	88.57	132.86	177.14
.2	44.3567	88.71	133.07	177.43
.3	44.4271	88.85	133.28	177.71
.4	44.4974	88.99	133.49	177.99
.5	44.5679	89.14	133.70	178.27
.6	44.6384	89.28	133.91	178.55
.7	44.7089	89.42	134.13	178.84
.8	44.7795	89.56	134.34	179.12
.9	44.8502	89.70	134.55	179.40
127.0	44.9209	89.84	134.76	179.68
.1	44.9916	89.98	134.98	179.97
.2	45.0625	90.12	135.19	180.25
.3	45.1334	90.27	135.40	180.53
.4	45.2043	90.41	135.61	180.82
.5	45.2753	90.55	135.83	181.10
.6	45.3463	90.69	136.04	181.38
.7	45.4174	90.83	136.25	181.67
.8	45.4886	90.98	136.46	181.95
.9	45.5598	91.12	136.68	182.24
128.0	45.6311	91.26	136.89	182.52
.1	45.7024	91.40	137.11	182.81
.2	45.7738	91.55	137.32	183.10
.3	45.8452	91.69	137.54	183.38
.4	45.9167	91.83	137.75	183.67
.5	45.9883	91.98	137.96	183.75
.6	46.0599	92.12	138.18	184.24
.7	46.1315	92.26	138.39	184.52
.8	46.2032	92.41	138.61	184.81
.9	46.2750	92.55	138.83	185.10

DEPTH.				
5	6	7	8	9
221.08	265.30	309.51	353.73	397.94
221.43	265.72	310.00	354.29	398.57
221.79	266.14	310.50	354.86	399.21
222.14	266.56	310.99	355.42	399.84
222.49	266.98	311.48	355.98	400.47
222.84	267.41	311.98	356.54	401.11
223.19	267.83	312.47	357.10	401.74
223.55	268.25	312.96	357.67	402.38
223.90	268.67	313.45	358.23	403.01
224.25	269.10	313.95	358.80	403.65
224.61	269.53	314.45	359.37	404.29
224.96	269.95	314.94	359.94	404.93
225.31	270.37	315.43	360.50	405.56
225.67	270.80	315.93	361.06	406.20
226.02	271.22	316.43	361.63	406.84
226.38	271.65	316.93	362.20	407.48
226.73	272.08	317.42	362.77	408.11
227.09	272.50	317.92	363.34	408.75
227.44	272.93	318.42	363.90	409.39
227.80	273.36	318.92	364.48	410.04
228.16	273.79	319.42	365.05	410.68
228.51	274.21	319.91	365.62	411.32
228.87	274.64	320.42	366.19	411.97
229.23	275.07	320.92	366.77	412.61
229.58	275.50	321.42	367.34	413.25
229.94	275.93	321.92	367.90	413.89
230.30	276.36	322.42	368.48	414.54
230.66	276.79	322.92	369.05	415.18
231.02	277.22	323.42	369.62	415.83
231.38	277.65	323.93	370.20	416.48

Diam. in Inches.	DEPTH.			
	1	2	3	4
129.0	46.3468	92.69	139.04	185.39
.1	46.4187	92.84	139.26	185.68
.2	46.4907	92.98	139.47	185.96
.3	46.5627	93.13	139.69	186.25
.4	46.6347	93.27	139.91	186.54
.5	46.7068	93.41	140.12	186.83
.6	46.7790	93.56	140.34	187.12
.7	46.8512	93.70	140.55	187.40
.8	46.9235	93.85	140.77	187.69
.9	46.9958	93.99	140.99	187.98
130.0	47.0682	94.14	141.20	188.27
.1	47.1406	94.28	141.42	188.56
.2	47.2131	94.43	141.64	188.85
.3	47.2857	94.57	141.86	189.14
.4	47.3583	94.72	142.07	189.43
.5	47.4309	94.86	142.29	189.72
.6	47.5037	95.01	142.51	190.02
.7	47.5764	95.15	142.73	190.30
.8	47.6493	95.30	142.95	190.60
.9	47.7222	95.44	143.17	190.89
131.0	47.7951	95.59	143.39	191.18
.1	47.8681	95.74	143.60	191.47
.2	47.9412	95.88	143.82	191.76
.3	48.0143	96.03	144.04	192.06
.4	48.0874	96.17	144.26	192.35
.5	48.1606	96.32	144.48	192.64
.6	48.2339	96.47	144.70	192.94
.7	48.3073	96.61	144.92	193.23
.8	48.3806	96.76	145.14	193.52
.9	48.4541	96.91	145.36	193.82

DEPTH.				
5	6	7	8	9
231.74	278.08	324.43	370.78	417.12
232.10	278.51	324.93	371.35	417.77
232.46	278.95	325.44	371.93	418.42
232.82	279.38	325.94	372.50	419.07
233.18	279.81	326.45	373.08	419.72
233.54	280.24	326.95	373.66	420.36
233.90	280.67	327.45	374.23	421.01
234.26	281.11	327.96	374.81	421.66
234.62	281.54	328.46	375.38	422.31
234.98	281.98	328.97	375.97	422.96
235.34	282.41	329.48	376.54	423.61
235.70	282.84	329.98	377.12	424.26
236.07	283.28	330.49	377.70	424.92
236.43	283.72	331.00	378.29	425.57
236.79	284.15	331.51	378.86	426.22
237.16	284.59	332.02	379.45	426.88
237.52	285.02	332.53	380.03	427.54
237.88	285.46	333.03	380.61	428.18
238.25	285.89	333.54	381.19	428.84
238.61	286.33	334.05	381.78	429.50
238.98	286.77	334.57	382.36	430.16
239.34	287.21	335.08	382.94	430.81
239.71	287.65	335.59	383.53	431.47
240.07	288.08	336.10	384.11	432.13
240.44	288.52	336.61	384.70	432.78
240.80	288.96	337.12	385.28	433.44
241.17	289.40	337.64	385.87	434.11
241.54	289.84	338.15	386.46	434.76
241.90	290.28	338.66	387.04	435.42
242.27	290.72	339.18	387.63	436.09

Diam. in Inches.	DEPTH.			
	1	2	3	4
132.0	48.5276	97.05	145.58	194.11
.1	48.6011	97.20	145.80	194.40
.2	48.6747	97.35	146.03	194.70
.3	48.7484	97.50	146.24	194.99
.4	48.8221	97.64	146.47	195.29
.5	48.8959	97.79	146.69	195.58
.6	48.9697	97.94	146.91	195.88
.7	49.0436	98.09	147.13	196.17
.8	49.1176	98.23	147.35	196.47
.9	49.1916	98.38	147.57	196.76
133.0	49.2656	98.53	147.80	197.06
.1	49.3397	98.68	148.02	197.36
.2	49.4139	98.83	148.24	197.66
.3	49.4881	98.98	148.46	197.95
.4	49.5624	99.12	148.69	198.25
.5	49.6367	99.27	148.91	198.55
.6	49.7111	99.42	149.13	198.84
.7	49.7856	99.57	149.36	199.14
.8	49.8601	99.72	149.58	199.44
.9	49.9346	99.87	149.80	199.74
134.0	50.0093	100.02	150.03	200.04
.1	50.0839	100.17	150.25	200.34
.2	50.1586	100.32	150.47	200.63
.3	50.2334	100.47	150.70	200.93
.4	50.3083	100.62	150.92	201.23
.5	50.3832	100.77	151.15	201.53
.6	50.4581	100.92	151.37	201.83
.7	50.5331	101.07	151.60	202.13
.8	50.6082	101.22	151.82	202.43
.9	50.6833	101.37	152.05	202.73

DEPTH.				
5	6	7	8	9
242.64	291.16	339.69	388.22	436.74
243.01	291.61	340.21	388.81	437.41
243.38	292.05	340.73	389.40	438.08
243.74	292.49	341.24	389.98	438.73
244.11	292.93	341.75	390.58	439.40
244.48	293.38	342.27	391.17	440.06
244.85	293.82	342.79	391.76	440.73
245.22	294.26	343.30	392.34	441.39
245.59	294.70	343.82	392.94	442.05
245.96	295.15	344.34	393.53	442.72
246.33	295.59	344.86	394.12	443.39
246.70	296.04	345.38	394.72	444.06
247.07	296.48	345.90	395.31	444.73
247.44	296.93	346.42	395.90	445.39
247.81	297.37	346.93	396.50	446.06
248.19	297.82	347.46	397.10	446.73
248.56	298.27	347.98	397.69	447.40
248.93	298.71	348.50	398.28	448.07
249.30	299.16	349.02	398.88	448.74
249.67	299.60	349.54	399.47	449.41
250.05	300.05	350.06	400.07	450.08
250.42	300.50	350.59	400.67	450.76
250.79	300.95	351.11	401.26	451.42
251.17	301.40	351.63	401.86	452.10
251.54	301.85	352.16	402.46	452.77
251.92	302.30	352.68	403.06	453.45
252.29	302.75	353.21	403.66	454.12
252.67	303.20	353.73	404.26	454.80
253.04	303.65	354.26	404.86	455.47
253.42	304.10	354.78	405.46	456.15

Diam. in Inches.	DEPTH.			
	1	2	3	4
135.0	50.7584	101.52	152.27	203.03
.1	50.8340	101.67	152.50	203.34
.2	50.9090	101.82	152.73	203.64
.3	50.9842	101.97	152.95	203.94
.4	51.0600	102.12	153.18	204.24
.5	51.1351	102.27	153.41	204.54
.6	51.2110	102.42	153.63	204.84
.7	51.2861	102.57	153.86	205.14
.8	51.3620	102.72	154.09	205.45
.9	51.4374	102.87	154.31	205.75
136.0	51.5132	103.03	154.54	206.05
.1	51.5890	103.18	154.77	206.36
.2	51.6650	103.33	155.00	206.66
.3	51.7410	103.48	155.22	206.96
.4	51.8170	103.63	155.45	207.27
.5	51.8930	103.79	155.68	207.57
.6	51.9690	103.94	155.91	207.88
.7	52.0450	104.09	156.14	208.18
.8	52.1210	104.24	156.36	208.48
.9	52.1972	104.39	156.59	208.79
137.0	52.2740	104.55	156.82	209.10
.1	52.3500	104.70	157.05	209.40
.2	52.4262	104.85	157.28	209.70
.3	52.5030	105.01	157.51	210.01
.4	52.5792	105.16	157.74	210.32
.5	52.6560	105.31	157.97	210.62
.6	52.7324	105.46	158.20	210.93
.7	52.8090	105.62	158.43	211.24
.8	52.8860	105.77	158.66	211.54
.9	52.9630	105.93	158.89	211.85

DEPTH.				
5	6	7	8	9
253.79	304.55	355.31	406.06	456.82
254.17	305.00	355.84	406.67	457.51
254.55	305.45	356.36	407.27	458.18
254.92	305.90	356.89	407.87	458.86
255.30	306.36	357.42	408.48	459.54
255.68	306.81	357.94	409.08	460.22
256.06	307.27	358.48	409.69	460.90
256.43	307.72	359.00	410.29	461.57
256.81	308.17	359.53	410.90	462.26
257.19	308.62	360.06	411.50	462.93
257.57	309.08	360.59	412.10	463.62
257.95	309.53	361.12	412.71	464.30
258.33	309.99	361.66	413.32	464.99
258.71	310.45	362.19	413.93	465.67
259.09	310.90	362.72	414.54	466.35
259.47	311.36	363.25	415.14	467.04
259.85	311.81	363.78	415.75	467.72
260.23	312.27	364.32	416.36	468.41
260.61	312.73	364.85	416.97	469.09
260.99	313.18	365.38	417.58	469.77
261.37	313.64	365.92	418.19	470.47
261.75	314.10	366.45	418.80	471.15
262.13	314.56	366.98	419.41	471.83
262.52	315.02	367.52	420.02	472.53
262.90	315.47	368.05	420.63	473.21
263.28	315.94	368.59	421.25	473.90
263.66	316.39	369.12	421.86	474.59
264.05	316.85	369.66	422.47	475.28
264.43	317.32	370.20	423.09	475.97
264.82	317.78	370.74	423.70	476.67

Diam. in Inches.	DEPTH.			
	1	2	3	4
138.0	53.0394	106.08	159.12	212.16
.1	53.1164	106.23	159.35	212.46
.2	53.1933	106.39	159.58	212.77
.3	53.2703	106.54	159.81	213.08
.4	53.3474	106.69	160.04	213.39
.5	53.4245	106.85	160.27	213.70
.6	53.5017	107.00	160.51	214.01
.7	53.5789	107.16	160.74	214.32
.8	53.6562	107.31	160.97	214.62
.9	53.7335	107.47	161.20	214.93
139.0	53.8109	107.62	161.43	215.24
.1	53.8884	157.78	161.66	215.55
.2	53.9659	107.93	161.90	215.86
.3	54.0434	108.09	162.13	216.17
.4	54.1211	108.24	162.36	216.48
.5	54.1987	108.40	162.60	216.80
.6	54.2765	108.55	162.83	217.10
.7	54.3543	108.71	163.06	217.42
.8	54.4321	108.86	163.30	217.73
.9	54.5100	109.02	163.53	218.04
140.0	54.5880	109.18	163.76	218.35
.1	54.6660	109.33	164.00	218.66
.2	54.7440	109.49	164.23	218.98
.3	54.8222	109.64	164.47	219.29
.4	54.9003	109.80	164.70	219.60
.5	54.9786	109.96	164.93	219.91
.6	55.0569	110.11	165.17	220.23
.7	55.1352	110.27	165.41	220.54
.8	55.2136	110.43	165.64	220.85
.9	55.2921	110.58	165.88	221.17

DEPTH.				
5	6	7	8	9
265.20	318.23	371.27	424.31	477.35
265.58	318.70	371.81	424.93	478.04
265.97	319.16	372.35	425.54	478.74
266.35	319.62	372.89	426.16	479.43
266.74	320.08	373.43	426.78	480.12
267.12	320.54	373.97	427.39	480.82
267.51	321.01	374.51	428.02	481.52
267.90	321.48	375.05	428.63	482.21
268.28	321.94	375.59	429.25	482.90
268.67	322.40	376.13	429.86	483.60
269.06	322.87	376.68	430.49	484.30
269.44	323.33	377.22	431.10	484.99
269.83	323.80	377.76	431.73	485.69
270.22	324.26	378.30	432.34	486.39
270.61	324.73	378.85	432.97	487.09
271.00	325.19	379.39	433.59	487.79
271.38	325.66	379.93	434.21	488.48
271.77	326.12	380.48	434.83	489.19
272.16	326.59	381.02	435.46	489.89
272.55	327.06	381.57	436.08	490.59
272.94	327.53	382.12	436.70	491.29
273.33	328.00	382.66	437.33	491.99
273.72	328.46	383.21	437.95	492.70
274.11	328.93	383.75	438.58	493.40
274.50	329.40	384.30	439.20	494.10
274.89	329.87	384.85	439.82	494.80
275.29	330.34	385.40	440.46	495.51
275.68	330.81	385.95	441.08	496.22
276.07	331.28	386.49	441.70	496.92
276.46	331.75	387.04	442.34	497.63

Diam. in Inches.	DEPTH.			
	1	2	3	4
141.0	55.3706	110.74	166.11	221.48
.1	55.4491	110.90	166.35	221.80
.2	55.5278	111.06	166.58	222.11
.3	55.6064	111.21	166.82	222.42
.4	55.6852	111.37	167.06	222.74
.5	55.7640	111.53	167.29	223.06
.6	55.8428	111.69	167.53	223.37
.7	55.9217	111.84	167.77	223.69
.8	56.0007	112.00	168.00	224.00
.9	56.0797	112.16	168.24	224.32
142.0	56.1587	112.32	168.48	224.64
.1	56.2379	112.48	168.71	224.95
.2	56.3170	112.63	168.95	225.27
.3	56.3963	112.79	169.19	225.58
.4	56.4756	112.95	169.43	225.90
.5	56.5549	113.11	169.67	226.22
.6	56.6343	113.27	169.90	226.54
.7	56.7138	113.43	170.14	226.86
.8	56.7933	113.59	170.38	227.17
.9	56.8729	113.75	170.62	227.49
143.0	56.9525	113.90	170.86	227.81
.1	57.0322	114.06	171.10	228.13
.2	57.1119	114.22	171.34	228.45
.3	57.1917	114.38	171.58	228.77
.4	57.2716	114.54	171.81	229.08
.5	57.3515	114.70	172.05	229.40
.6	57.4314	114.86	172.29	229.72
.7	57.5114	115.02	172.53	230.04
.8	57.5915	115.18	172.77	230.36
.9	57.6716	115.34	173.01	230.68

DEPTH.				
5	6	7	8	9
276.85	332.22	387.59	442.96	498.33
277.25	332.69	388.14	443.59	499.04
277.64	333.17	388.70	444.22	499.75
278.03	333.64	389.24	444.85	500.45
278.43	334.11	389.80	445.48	501.17
278.82	334.58	390.35	446.11	501.88
279.22	335.06	390.90	446.74	502.59
279.61	335.53	391.45	447.38	503.30
280.01	336.01	392.01	448.01	504.01
280.40	336.48	392.56	448.64	504.72
280.80	336.95	393.11	449.27	505.43
281.19	337.43	393.67	449.90	506.14
281.59	337.90	394.22	450.54	506.85
281.98	338.38	394.77	451.17	507.56
282.38	338.85	395.33	451.80	508.28
282.78	339.33	395.89	452.44	509.00
283.17	339.80	396.44	453.07	509.71
283.57	340.28	397.00	453.71	510.43
283.97	340.76	397.55	454.34	511.14
284.37	341.24	398.11	454.98	511.86
284.76	341.71	398.66	455.62	512.57
285.16	342.19	399.22	456.26	513.29
285.56	342.67	399.78	456.90	514.01
285.96	343.15	400.34	457.54	514.73
286.36	343.63	400.90	458.17	515.44
286.76	344.11	401.46	458.81	516.16
287.16	344.59	402.02	459.45	516.88
287.56	345.07	402.58	460.09	517.60
287.96	345.55	403.14	460.73	518.32
288.36	346.03	403.70	461.37	519.04

Diam. in Inches.	DEPTH.			
	1	2	3	4
144.0	57.7518	115.50	173.26	231.01
.1	57.8321	115.66	173.50	231.33
.2	57.9124	115.82	173.74	231.65
.3	57.9927	115.99	173.98	231.97
.4	58.0731	116.15	174.22	232.29
.5	58.1536	116.31	174.46	232.61
.6	58.2341	116.47	174.70	232.94
.7	58.3147	116.63	174.95	233.26
.8	58.3953	116.79	175.19	233.58
.9	58.4760	116.95	175.43	233.90
145.0	58.5567	117.11	175.67	234.23
.1	58.6375	117.27	175.91	234.55
.2	58.7184	117.44	176.15	234.87
.3	58.7993	117.60	176.40	235.20
.4	58.8802	117.76	176.64	235.52
.5	58.9613	117.92	176.88	235.84
.6	59.0423	118.08	177.13	236.17
.7	59.1235	118.25	177.37	236.49
.8	59.2047	118.41	177.62	236.82
.9	59.2859	118.57	177.86	237.14
146.0	59.3672	118.73	178.10	237.47
.1	59.4485	118.90	178.34	237.79
.2	59.5299	119.06	178.59	238.12
.3	59.6114	119.22	178.83	238.44
.4	59.6929	119.39	179.08	238.77
.5	59.7745	119.55	179.32	239.10
.6	59.8561	119.71	179.57	239.42
.7	59.9378	119.88	179.81	239.75
.8	60.0196	120.04	180.06	240.08
.9	60.1014	120.20	180.30	240.40

DEPTH.				
5	6	7	8	9
288.76	346.51	404.26	462.02	519.77
289.16	346.99	404.82	462.66	520.49
289.56	347.47	405.38	463.30	521.21
289.97	347.96	405.95	463.94	521.94
290.37	348.44	406.51	464.58	522.66
290.77	348.92	407.07	465.22	523.38
291.17	349.40	407.64	465.87	524.11
291.58	349.89	408.21	466.52	524.84
291.98	350.37	408.77	467.16	525.56
292.38	350.86	409.33	467.81	526.28
292.79	351.34	409.90	468.46	527.01
293.19	351.82	410.48	469.10	527.73
293.59	352.31	411.03	469.74	528.46
294.00	352.79	411.59	470.39	529.19
294.40	353.28	412.16	471.04	529.92
294.81	353.77	412.73	471.69	530.65
295.21	354.25	413.29	472.34	531.38
295.62	354.74	413.86	472.98	532.11
296.03	355.23	414.44	473.64	532.85
296.43	355.72	415.00	474.29	533.57
296.84	356.20	415.57	474.94	534.30
297.24	356.69	416.14	475.58	535.03
297.65	357.18	416.71	476.24	535.77
298.06	357.67	417.28	476.89	536.50
298.47	358.16	417.85	477.54	537.24
298.87	358.64	418.42	478.19	537.97
299.28	359.14	418.99	478.85	538.70
299.69	359.63	419.57	479.50	539.44
300.10	360.11	420.13	480.15	540.17
300.51	360.61	420.71	480.81	540.91

Diam. in Inches.	DEPTH.			
	1	2	3	4
147.0	60.1832	120.37	180.55	240.73
.1	60.2651	120.53	180.80	241.06
.2	60.3471	120.69	181.04	241.39
.3	60.4291	120.86	181.29	241.72
.4	60.5112	121.02	181.53	242.04
.5	60.5933	121.19	181.78	242.37
.6	60.6755	121.35	182.03	242.70
.7	60.7578	121.51	182.27	243.03
.8	60.8401	121.68	182.52	243.36
.9	60.9224	121.84	182.77	243.69
148.0	61.0048	122.01	183.01	244.02
.1	61.0873	122.17	183.26	244.35
.2	61.1698	122.34	183.51	244.68
.3	61.2524	122.50	183.76	245.01
.4	61.3350	122.67	184.01	245.34
.5	61.4177	122.83	184.25	245.67
.6	61.5005	123.00	184.50	246.00
.7	61.5833	123.17	184.75	246.33
.8	61.6661	123.33	185.00	246.66
.9	61.7490	123.50	185.25	247.00
149.0	61.8320	123.66	185.50	247.33
.1	61.9150	123.83	185.75	247.66
.2	61.9981	124.00	185.99	247.99
.3	62.0812	124.16	186.24	248.32
.4	62.1644	124.33	186.49	248.66
.5	62.2477	124.49	186.74	248.99
.6	62.3310	124.66	186.99	249.32
.7	62.4143	124.83	187.24	249.66
.8	62.4978	124.99	187.49	249.99
.9	62.5812	125.16	187.74	250.32

DEPTH.				
5	6	7	8	9
300.92	361.10	421.28	481.46	541.65
301.33	361.59	421.86	482.12	542.39
301.74	362.08	422.43	482.78	543.12
302.15	362.57	423.00	483.43	543.86
302.56	363.07	423.58	484.09	544.60
302.96	363.56	424.15	484.74	545.34
303.38	364.05	424.73	485.40	546.08
303.79	364.54	425.30	486.06	546.81
304.20	365.04	425.88	486.72	547.56
304.61	365.53	426.45	487.38	548.30
305.02	366.02	427.03	488.03	549.04
305.44	366.52	427.61	488.70	549.78
305.85	367.01	428.18	489.35	550.52
306.26	367.51	428.76	490.02	551.26
306.68	368.01	429.35	490.68	552.02
307.09	368.50	429.92	491.34	552.75
307.50	369.00	430.50	492.00	553.50
307.92	369.50	431.08	492.66	554.25
308.33	370.00	431.66	493.33	554.99
308.75	370.49	432.24	493.99	555.74
309.16	370.99	432.82	494.66	556.49
309.58	371.49	433.41	495.32	557.24
309.99	371.99	433.99	495.98	557.98
310.41	372.49	434.57	496.65	558.73
310.82	372.98	435.15	497.31	559.48
311.24	373.48	435.73	497.98	560.22
311.66	373.99	436.32	498.65	560.98
312.07	374.48	436.90	499.31	561.73
312.49	374.98	437.48	499.98	562.47
312.91	375.49	438.07	500.65	563.23

Diam. in Inches.	DEPTH.			
	1	2	3	4
150.0	62.6648	125.33	187.99	250.66
.1	62.7483	125.50	188.24	250.99
.2	62.8320	125.66	188.50	251.33
.3	62.9157	125.83	188.75	251.66
.4	62.9994	126.00	189.00	252.00
.5	63.0832	126.17	189.25	252.33
.6	63.1671	126.33	189.50	252.67
.7	63.2510	126.50	189.75	253.00
.8	63.3350	126.67	190.01	253.34
.9	63.4190	126.84	190.26	253.68
151.0	63.5030	127.01	190.51	254.01
.1	63.5872	127.17	190.76	254.35
.2	63.6714	127.34	191.01	254.68
.3	63.7556	127.51	191.27	255.02
.4	63.8400	127.68	191.52	255.36
.5	63.9243	127.85	191.77	255.70
.6	64.0087	128.02	192.02	256.03
.7	64.0932	128.19	192.28	256.37
.8	64.1777	128.35	192.53	256.71
.9	64.2623	128.52	192.79	257.05
152.0	64.3470	128.69	193.04	257.39
.1	64.4316	128.86	193.29	257.72
.2	64.5164	129.03	193.55	258.06
.3	64.6012	129.20	193.80	258.40
.4	64.6861	129.37	194.06	258.74
.5	64.7710	129.54	194.31	259.08
.6	64.8560	129.71	194.57	259.42
.7	64.9410	129.88	194.82	259.76
.8	65.0261	130.05	195.08	260.10
.9	65.1112	130.22	195.33	260.44

DEPTH.				
5	6	7	8	9
313.32	375.98	438.65	501.31	563.98
313.74	376.49	439.24	501.98	564.73
314.16	376.99	439.82	502.66	565.49
314.58	377.49	440.41	503.32	566.24
315.00	377.89	440.89	503.89	566.99
315.42	378.50	441.58	504.66	567.75
315.84	379.00	442.17	505.34	568.50
316.26	379.51	442.76	506.01	569.26
316.68	380.01	443.35	506.68	570.02
317.10	380.51	443.93	507.35	570.77
317.52	381.02	444.52	508.02	571.53
317.94	381.52	445.11	508.70	572.28
318.36	382.03	445.70	509.37	573.04
318.78	382.53	446.29	510.04	573.80
319.20	383.04	446.88	510.72	574.56
319.62	383.54	447.47	511.39	575.32
320.04	384.05	448.06	512.06	576.07
320.47	384.56	448.65	512.74	576.84
320.89	385.06	449.24	513.42	577.59
321.31	385.57	449.83	514.10	578.36
321.74	386.08	450.43	514.78	579.12
322.16	386.59	451.02	515.45	579.88
322.58	387.10	451.61	516.13	580.64
323.01	387.61	452.21	516.81	581.41
323.43	388.12	452.80	517.49	582.17
323.86	388.63	453.40	518.17	582.94
324.28	389.14	453.99	518.85	583.70
324.71	389.65	454.59	519.53	584.47
325.13	390.16	455.18	520.21	585.23
325.56	390.67	455.78	520.89	586.00

Diam. in Inches.	DEPTH.			
	1	2	3	4
153.0	65.1964	130.39	195.59	260.78
.1	65.2817	130.56	195.84	261.12
.2	65.3670	130.73	196.10	261.47
.3	65.4523	130.90	196.36	261.81
.4	65.5377	131.07	196.61	262.15
.5	65.6232	131.25	196.87	262.49
.6	65.7088	131.42	197.12	262.83
.7	65.7943	131.59	197.38	263.18
.8	65.8800	131.76	197.64	263.52
.9	65.9657	131.93	197.90	263.86
154.0	66.0514	132.10	198.15	264.20
.1	66.1372	132.27	198.41	264.55
.2	66.2231	132.45	198.67	264.89
.3	66.3090	132.62	198.93	265.24
.4	66.3950	132.79	199.19	265.58
.5	66.4810	133.96	199.44	265.92
.6	66.5671	133.13	199.70	266.27
.7	66.6533	133.31	199.96	266.61
.8	66.7395	133.48	200.22	266.96
.9	66.8257	133.65	200.48	267.30
155.0	66.9120	133.82	200.74	267.65
.1	66.9984	134.00	200.99	267.99
.2	67.0848	134.17	201.25	268.34
.3	67.1713	134.34	201.51	268.68
.4	67.2578	134.51	201.77	269.03
.5	67.3444	134.69	202.03	269.38
.6	67.4311	134.86	202.29	269.72
.7	67.5178	135.03	202.55	270.07
.8	67.6045	135.21	202.81	270.42
.9	67.6013	135.38	203.07	270.76

DEPTH.				
5	6	7	8	9
325.98	391.18	456.37	521.57	586.76
326.41	391.69	456.97	522.25	587.53
326.84	392.20	457.57	522.94	588.30
327.26	392.71	458.16	523.62	589.07
327.69	393.22	458.76	524.30	589.83
328.12	393.73	459.36	524.98	590.61
328.54	394.24	459.96	525.66	591.37
328.97	394.76	460.56	526.35	592.15
329.40	395.28	461.16	527.04	592.92
329.83	395.79	461.76	527.72	593.69
330.26	396.31	462.36	528.41	594.46
330.69	396.82	462.96	529.10	595.23
331.12	397.35	463.56	529.78	596.01
331.55	397.85	464.16	530.47	596.78
331.98	398.37	464.77	531.16	597.56
332.41	398.89	465.37	531.85	598.33
332.84	399.40	465.97	532.54	599.10
333.27	399.92	466.57	533.22	599.88
333.70	400.43	467.17	533.91	600.65
334.13	400.95	467.78	534.60	601.43
334.56	401.47	468.38	535.30	602.21
334.99	401.99	468.99	535.98	602.98
335.42	402.50	469.59	536.67	603.76
335.86	403.03	470.20	537.37	604.54
336.29	403.54	470.80	538.06	605.31
336.72	404.06	471.41	538.75	606.10
337.16	404.59	472.02	539.45	606.88
337.59	405.10	472.62	540.14	607.65
338.02	405.62	473.23	540.83	608.44
338.46	406.15	473.84	541.53	609.22

Excise for Common Brewers in the Country at 4 *l.* 9. *per* Barrel, the Allowances being 2 ½ Barrels in every 23.

B.	0					¼					½					¾				
	l.	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	4	2	3	5	0	5	3	2		0	6	4	0	19	0	7	4	3	14
2	0	8	5	2	10	0	9	6	1	5	0	10	7	0	1	0	11	7	2	19
3	0	12	8	1	15	0	13	9	0	10	0	14	9	3	6	0	15	10	2	1
4	0	16	11	0	20	0	17	11	3	15	0	19	0	2	11	1	0	1	1	6
5	1	1	2	0	2	1	2	2	2	20	1	3	3	1	16	1	4	4	0	11
6	1	5	4	3	7	1	6	5	2	2	1	7	6	0	21	1	8	6	3	16
7	1	9	7	2	12	1	10	8	1	7	1	11	9	0	3	1	12	9	2	21
8	1	13	10	1	17	1	14	11	0	12	1	15	11	3	8	1	17	0	2	3
9	1	11	1	0	22	1	19	1	3	17	2	0	2	2	13	2	1	3	1	8
10	2	2	4	0	4	2	3	4	2	22	2	4	5	1	18	2	5	6	0	13
11	2	6	6	3	9	2	7	7	2	4	2	8	8	1	0	2	9	8	3	18
12	2	10	9	2	14	2	11	10	1	9	2	12	11	0	5	2	13	11	3	0
13	2	15	0	1	19	2	16	1	0	14	2	17	1	3	10	2	18	2	2	5
14	3	19	3	1	1	3	0	3	3	19	3	1	4	2	15	3	2	5	1	10
15	3	3	6	0	6	3	4	6	2	1	3	5	7	1	20	3	6	8	0	15
16	3	7	8	3	11	3	8	9	2	6	3	9	10	1	2	3	10	10	3	20
17	3	11	11	2	16	3	13	0	1	11	3	14	1	0	7	3	15	1	3	2
18	3	16	2	1	21	3	17	3	0	16	3	18	3	3	12	3	19	4	2	7
19	4	0	5	1	3	4	1	5	3	21	4	2	6	2	17	4	3	7	1	12
20	4	4	8	0	8	4	5	8	3	3	4	6	9	1	22	4	7	10	0	17
21	4	8	10	3	13	4	9	11	2	8	4	11	0	1	4	4	12	0	3	22
22	4	13	1	2	18	4	14	2	1	13	4	15	3	0	9	4	16	3	3	4

Excise for Common Brewers in the Country, at 4 s. 9 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
23	4	17	4	2	713	150	18	7	2
46	9	14	9	0	736	155	16	0	0
69	14	12	1	2	759	160	13	4	2
92	19	9	6	0	782	165	10	9	0
115	24	6	10	2	805	170	8	1	2
138	29	4	3	0	828	175	5	6	0
161	34	1	7	2	851	180	2	10	2
184	38	19	0	0	874	185	0	3	0
207	43	16	4	2	897	189	17	7	2
230	48	13	9	0	920	194	15	0	0
253	53	11	1	2	943	199	12	4	2
276	58	8	6	0	966	204	9	9	0
299	63	5	10	2	989	209	7	1	2
322	68	3	3	0	1012	214	4	6	0
345	73	0	7	2	1035	219	1	10	2
368	77	18	0	0	1058	223	19	3	0
391	82	15	4	2	1081	228	16	7	2
414	87	12	9	0	1104	233	14	0	0
437	92	10	1	2	1127	238	11	4	2
460	97	7	6	0	1150	243	8	9	0
483	102	4	10	2	1173	248	6	1	2
506	107	2	3	0	1196	253	3	6	0
529	111	19	7	2	1219	258	0	10	2
552	116	17	0	0	1242	262	18	3	0
575	121	14	4	2	1265	267	15	7	2
598	126	11	9	0	1288	272	13	0	0
621	131	9	1	2	1311	277	10	4	2
644	136	6	6	0	1334	282	7	9	0
667	141	3	10	2	1357	287	5	1	2
690	146	1	3	0	1380	292	2	6	0

Excise for Common Brewers in the Country, at 4 s. 9 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	296	19	10	2	2093	443	1	1	2
1426	301	17	3	0	2116	447	18	6	0
1449	306	14	7	2	2139	452	15	10	2
1472	311	12	0	0	2162	457	13	3	0
1495	316	9	4	2	2185	462	10	7	2
1518	321	6	9	0	2208	467	8	0	0
1541	326	4	1	2	2231	472	5	4	2
1564	331	1	6	0	2254	477	2	9	0
1587	335	18	10	2	2277	482	0	1	2
1610	340	16	3	0	2300	486	17	6	0
1633	345	13	7	2	2323	491	14	10	2
1656	350	11	0	0	2346	496	12	3	0
1679	355	8	4	2	2369	501	9	7	2
1702	360	5	9	0	2392	506	7	0	0
1725	365	3	1	2	2415	511	4	4	2
1748	370	0	6	0	2438	516	1	9	0
1771	374	17	10	2	2461	520	19	1	2
1794	379	15	3	0	2484	525	16	6	0
1817	384	12	7	2	2507	530	13	10	2
1840	389	10	0	0	2530	535	11	3	0
1863	394	7	4	2	5060	1071	2	6	0
1886	399	4	9	0	7590	1606	13	9	0
1909	404	2	1	2	10120	2142	5	0	0
1932	408	19	6	0	12650	2677	16	3	0
1955	413	16	10	2	15180	3213	7	6	0
1978	418	14	3	0	17710	3748	18	9	0
2001	423	11	7	2	20240	4284	10	0	0
2024	428	9	0	0	22770	4820	1	3	0
2047	433	6	4	2	25300	5355	12	6	0
2070	438	3	9	0	27830	5891	3	9	0

Excise for Common Brewers in the Country at 1 s. 3 d. per Barrel, the Allowances being 2 ½ Barrels in every 23.

	0					¼					½					¾				
B.	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23
1	0	3	1	1	11	0	1	4	2	19	0	1	8	0	5	0	1	11	1	13
2	0	2	2	2	22	0	2	6	0	7	0	2	9	1	16	0	3	0	3	1
3	0	3	4	0	10	0	3	7	1	18	0	3	10	3	4	0	4	2	0	12
4	0	4	5	1	21	0	4	8	3	6	0	5	0	0	15	0	5	3	2	0
5	0	5	6	3	9	0	5	10	0	17	0	6	1	2	3	0	6	4	3	10
6	0	6	8	0	20	0	6	11	2	5	0	7	2	3	14	0	7	6	0	22
7	0	7	9	2	8	0	8	0	3	16	0	8	4	1	2	0	8	7	2	10
8	0	8	10	3	19	0	9	2	1	4	0	9	5	2	13	0	9	8	3	21
9	0	10	0	1	7	0	10	3	2	15	0	10	7	0	1	0	10	10	1	9
10	0	11	1	2	18	0	11	5	0	3	0	11	8	1	12	0	11	11	2	20
11	0	12	3	0	6	0	12	6	1	14	0	12	9	3	0	0	13	1	0	8
12	0	13	4	1	17	0	13	7	3	2	0	13	11	0	11	0	14	2	1	19
13	0	14	5	3	5	0	14	9	0	13	0	15	0	1	22	0	15	3	3	7
14	0	15	7	0	16	0	15	10	2	1	0	16	1	3	10	0	16	5	0	18
15	0	16	8	2	4	0	16	11	3	12	0	17	3	0	2	0	17	6	2	6
16	0	17	9	3	15	0	18	1	1	0	0	18	4	2	5	0	18	7	3	17
17	0	18	11	1	3	0	19	2	2	11	0	19	5	3	20	0	19	9	1	5
18	1	0	0	2	14	1	0	3	3	22	1	0	7	1	8	1	0	10	2	16
19	1	1	2	0	2	1	1	5	1	10	1	1	8	2	15	1	2	0	0	4
20	1	2	3	1	13	1	2	6	2	21	1	2	10	0	7	1	3	1	1	15
21	1	3	4	3	1	1	3	8	0	9	1	3	11	1	18	1	4	2	3	3
22	1	4	6	0	12	1	4	9	1	20	1	5	0	3	6	1	5	4	0	14

Excise

Excise for Common Brewers in the Country, at 1 *s.* 3 *d.* *per* Barrel, *&c.*

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f*
23	1	5	7	2	713	39	14	4	2
46	2	11	3	0	736	41	0	0	0
69	3	16	10	2	759	42	5	7	2
92	5	2	6	0	782	43	11	3	0
115	6	8	1	2	805	44	16	10	2
138	7	13	9	0	828	46	2	6	0
161	8	19	4	2	851	47	8	1	2
184	10	5	0	0	874	48	13	9	0
207	11	10	7	2	897	49	19	4	2
230	12	16	3	0	920	51	5	0	0
253	14	1	10	2	943	52	10	7	2
276	15	7	6	0	966	53	16	3	0
299	16	13	1	2	989	55	1	10	2
322	17	18	9	0	1012	56	7	6	0
345	19	4	4	2	1035	57	13	1	2
368	20	10	0	0	1058	58	18	9	0
391	21	15	7	2	1081	60	4	4	2
414	23	1	3	0	1104	61	10	0	0
437	24	6	10	2	1127	62	15	7	2
460	25	12	6	0	1150	64	1	3	0
483	26	18	1	2	1173	65	6	10	2
506	28	3	9	0	1196	66	12	6	0
529	29	9	4	2	1219	67	18	1	2
552	30	15	0	0	1242	69	3	9	0
575	32	0	7	2	1265	70	9	4	2
598	33	6	3	0	1288	71	15	0	0
621	34	11	10	2	1311	73	0	7	2
644	35	17	6	0	1334	74	6	3	0
667	37	3	1	2	1357	75	11	10	2
690	38	8	9	0	1380	76	17	6	0

Excise for Common Brewers in the Country, at 1 s. 3 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	78	3	1	2	2093	116	11	10	2
1426	79	8	9	0	2116	117	17	6	0
1449	80	14	4	2	2139	119	3	1	2
1472	82	0	0	0	2162	120	8	9	0
1495	83	5	7	2	2185	121	14	4	2
1518	84	11	3	0	2208	123	0	0	0
1541	85	16	10	2	2231	124	5	7	2
1564	87	2	6	0	2254	125	11	3	0
1587	88	8	1	2	2277	126	16	10	2
1610	89	13	9	0	2300	128	2	6	0
1633	90	19	4	2	2323	129	8	1	2
1656	92	5	0	0	2346	130	13	9	0
1679	93	10	7	2	2369	131	19	4	2
1702	94	16	3	0	2392	133	5	0	0
1725	96	1	10	2	2415	134	10	7	2
1748	97	7	6	0	2438	135	16	3	0
1771	98	13	1	2	2461	137	1	10	2
1794	99	18	9	0	2484	138	7	6	0
1817	101	4	4	2	2507	139	13	1	2
1840	102	10	0	0	2530	140	18	9	0
1863	103	15	7	2	5060	281	17	6	0
1886	105	1	3	0	7590	422	16	3	0
1909	106	6	10	2	10120	563	15	0	0
1932	107	12	6	0	12650	704	13	9	0
1955	108	18	1	2	15180	845	12	6	0
1978	110	3	9	0	17710	986	11	3	0
2001	111	9	4	2	20240	1127	10	0	0
2024	112	15	0	0	22770	1268	8	9	0
2047	114	0	7	2	25300	1409	7	6	0
2070	115	6	3	0	27830	1550	6	3	0

Excise for Common Brewers in *London*, at 4 *s.* 9 *d. per* Barrel, the Allowances being 3 Barrels in every 23 of *Beer*.

	0				¼				½				¾			
Bar.	*l.*	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22
1	0	4	1	2 6	0	5	1	3 15	0	6	2	1 9	0	7	2	2 22
2	0	8	3	0 12	0	9	3	2 2	0	10	3	3 15	0	11	4	1 5
3	0	12	4	2 18	0	13	5	0 8	0	14	5	1 21	0	15	5	3 11
4	0	16	6	1 1	0	17	6	2 14	0	18	7	0 4	0	19	7	1 17
5	1	0	7	3 7	1	1	8	0 20	1	2	8	2 10	1	3	9	0 0
6	1	4	9	1 13	1	5	9	3 3	1	6	10	0 16	1	7	10	2 6
7	1	8	10	3 19	1	9	11	1 9	1	10	11	2 22	1	12	0	0 12
8	1	13	0	2 2	1	14	0	3 15	1	15	1	1 5	1	16	1	2 18
9	1	17	2	0 8	1	18	2	1 21	1	19	2	3 11	2	0	3	1 1
10	2	1	3	2 14	2	2	4	0 4	2	3	4	1 17	2	4	4	3 7
11	2	5	5	0 20	2	6	5	2 10	2	7	6	0 0	2	8	6	1 13
12	2	9	6	3 3	2	10	7	0 16	2	11	7	2 6	2	12	7	3 19
13	2	13	8	1 9	2	14	8	2 22	2	15	9	0 12	2	16	9	2 2
14	2	17	9	3 15	2	18	10	1 5	2	19	10	2 18	3	0	11	0 8
15	3	1	11	1 21	3	2	11	3 11	3	4	0	1 1	3	5	0	2 14
16	3	6	1	0 4	3	7	1	1 17	3	8	1	3 7	3	9	2	0 20
17	3	10	2	2 10	3	11	3	0 0	3	12	3	1 13	3	13	3	3 3
18	3	14	4	0 16	3	15	4	2 6	3	16	4	3 15	3	17	5	1 9
19	3	18	5	2 22	3	19	6	0 12	4	0	6	2 2	4	1	6	3 15
20	4	2	7	1 5	4	3	7	2 18	4	4	8	0 8	4	5	8	1 21
21	4	6	8	3 11	4	7	9	1 1	4	8	9	2 14	4	9	10	0 4
22	4	10	10	1 17	4	11	10	3 7	4	12	11	0 20	4	13	11	2 10

Excise

Excise for Common Brewers in London, at 4 l. 9 d. per Bar. &c. Beer.

Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.
23	4	15	713	142	5	1403	289	15	2093	432	5
46	9	10	736	152	0	1426	294	10	2116	437	0
69	14	5	759	156	15	1449	299	5	2139	441	15
92	19	0	782	161	10	1472	304	0	2162	446	10
115	23	15	805	166	5	1495	308	15	2185	451	5
138	28	10	828	171	0	1518	313	10	2208	456	0
161	33	5	851	175	15	1541	318	5	2231	460	15
184	38	0	874	180	10	1564	323	0	2254	465	10
207	42	15	897	185	5	1587	327	15	2277	470	5
230	47	10	820	190	0	1610	332	10	2300	475	0
253	52	5	943	194	15	1633	337	5	2323	479	15
276	57	0	966	199	10	1656	342	0	2346	484	10
299	61	15	989	204	5	1679	346	15	2369	489	5
322	66	10	1012	209	0	1702	351	10	2392	494	0
345	71	5	1033	213	15	1725	356	5	2415	498	15
368	76	0	1058	218	10	1748	361	0	2438	503	10
391	80	15	1081	223	5	1771	365	15	2461	508	5
414	85	10	1104	228	0	1794	370	10	2484	513	0
437	90	5	1127	232	15	1817	375	5	2507	517	15
460	95	0	1150	237	10	1840	380	0	2530	522	10
483	99	15	1173	242	5	1863	384	15	5060	1045	0
506	104	10	1196	247	0	1886	389	10	7590	1567	10
529	109	5	1219	251	15	1909	394	5	10120	2090	0
552	114	0	1242	256	10	1932	399	0	12650	2612	10
575	118	15	1265	261	5	1955	403	15	15180	3135	0
598	123	10	1288	266	0	1978	408	10	17710	3657	10
621	128	5	1311	270	15	2001	413	5	20240	4180	0
644	133	0	1334	275	10	2024	418	0	22770	4702	10
667	137	15	1357	280	5	2047	422	15	25300	5225	0
690	142	10	1380	285	0	2070	427	10	27830	5747	10

Excise for Common Brewers in *London*, at 4 *s.* 9 *d.* per Barrel, the Allowances being 2 Barrels in every 22 of *Ale*.

Bar.	0					¼					½					¾				
	l.	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22
1	0	4	3	3	6	0	5	4	3	2	0	6	5	2	20	0	7	6	2	16
2	0	8	7	2	12	0	9	8	2	8	0	10	9	2	4	0	11	10	2	0
3	0	12	11	1	18	0	14	0	1	14	0	15	1	1	10	0	16	2	1	6
4	0	17	3	1	2	0	18	4	0	20	0	19	5	0	16	1	0	6	0	12
5	1	1	7	0	8	1	2	8	0	4	1	3	9	0	0	1	4	9	3	18
6	1	5	10	3	14	1	6	11	3	10	1	8	0	3	6	1	9	1	3	2
7	1	10	2	2	20	1	11	3	2	16	1	12	4	2	12	1	13	5	2	8
8	1	14	6	2	4	1	15	7	2	0	1	16	8	1	18	1	17	9	1	14
9	1	18	10	1	10	1	19	11	1	6	2	1	0	1	2	2	2	1	0	20
10	2	3	2	0	16	2	4	3	0	12	2	5	4	0	8	2	6	5	0	4
11	2	7	6	0	0	2	8	6	3	18	2	9	7	3	14	2	10	8	3	10
12	2	11	9	3	6	2	12	10	3	2	2	13	11	2	20	2	15	0	2	16
13	2	16	1	2	12	2	17	2	2	8	2	18	3	2	4	2	19	4	2	0
14	3	0	5	1	18	3	1	6	1	14	3	2	7	1	10	3	3	8	1	6
15	3	4	9	1	2	3	5	10	0	20	3	6	11	0	16	3	8	0	0	12
16	3	9	1	0	8	3	10	2	0	4	3	11	3	0	0	3	12	3	3	18
17	3	13	4	3	14	3	14	5	3	10	3	15	6	3	6	3	16	7	3	2
18	3	17	8	2	20	3	18	9	2	16	3	19	10	2	12	4	0	11	2	8
19	4	2	0	2	4	4	3	1	2	0	4	4	2	1	18	4	5	3	1	14
20	4	6	4	1	10	4	7	5	1	6	4	8	6	1	2	4	9	7	0	20
22	4	10	8	0	16	4	11	9	0	12	4	12	10	0	8	4	13	11	0	4

Exciſe for Common Brewers in *London*, at 4 *s.* 9 *d.* *per* Bar. *&c. Ale.*

Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar	*l.*	*s.*	Bar.	*l.*	*s.*
22	4	15	682	147	5	1342	289	15	2002	432	5
44	9	10	704	152	0	1364	294	10	2024	437	0
66	14	5	726	156	15	1386	299	5	2046	441	15
88	19	0	748	161	10	1408	304	0	2068	446	10
110	23	15	770	166	5	1430	308	15	2090	451	5
132	28	10	792	171	0	1452	313	10	2112	456	0
154	33	5	81.	175	15	1474	318	5	2134	460	15
176	33	0	836	180	10	1496	323	0	2156	465	10
198	42	15	858	185	5	1518	327	15	2178	470	5
220	47	10	880	190	0	1540	332	10	2200	475	0
242	52	5	902	194	15	1562	337	5	2222	479	15
264	57	0	924	199	10	1584	342	0	2244	484	10
286	61	15	946	204	5	1606	346	15	2266	489	5
308	66	10	968	209	0	1628	351	10	2288	494	0
330	71	5	990	213	15	1650	356	5	2310	498	15
352	76	0	1012	218	10	1672	361	0	2332	503	10
374	80	15	1034	223	5	1694	365	15	2354	508	5
396	85	10	1056	228	0	1716	370	10	2376	513	0
418	90	5	1078	232	15	1738	375	5	2398	517	15
440	95	0	1100	237	10	1760	380	0	2420	522	10
462	99	15	1122	242	5	1782	384	15	4840	1045	0
484	104	10	1144	247	0	1804	389	10	7260	1567	10
506	109	5	1166	251	15	1826	394	5	9680	2090	0
528	114	0	1188	256	10	1848	399	0	12100	2612	10
550	118	15	1210	261	5	1870	403	15	14520	3135	0
572	123	10	1232	266	0	1892	408	10	16940	3657	10
594	128	5	1254	270	15	1914	413	5	19360	4180	0
616	133	0	1276	275	10	1936	418	0	21780	4702	10
638	137	15	1298	280	5	1958	422	15	24200	5225	0
660	142	10	1320	285	0	1980	427	10	26620	5747	10

Excise for Common Brewers in *London*, at 1 *s.* 3 *d. per* Barrel, the Allowances being 3 Bar. in every 23 *Small Beer.*

	0				¼				½				¾			
Bar.	*l.*	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23
1	0	1	1	0 4	0	1	4	1 5	0	1	7	2 6	0	1	10	3 7
2	0	2	2	0 8	0	2	5	1 9	0	2	8	2 10	0	2	11	3 11
3	0	3	3	0 12	0	3	6	1 13	0	3	9	2 14	0	4	0	3 15
4	0	4	4	0 16	0	4	7	1 17	0	4	10	2 18	0	5	1	3 19
5	0	5	5	0 20	0	5	8	1 21	0	5	11	2 22	0	6	3	0 0
6	0	6	6	1 1	0	6	9	2 2	0	7	0	3 3	0	7	4	0 4
7	0	7	7	1 5	0	7	10	2 6	0	8	1	3 7	0	8	5	0 8
8	0	8	8	1 9	0	8	11	2 10	0	9	2	3 11	0	9	6	0 12
9	0	9	9	1 13	0	10	0	2 14	0	10	3	3 15	0	10	7	0 16
10	0	10	10	1 17	0	11	1	2 18	0	11	4	3 19	0	11	8	0 20
11	0	11	11	1 21	0	12	2	2 22	0	12	6	0 0	0	12	9	1 1
12	0	13	0	2 2	0	13	3	3 3	0	13	7	0 4	0	13	10	1 5
13	0	14	1	2 6	0	14	4	3 7	0	14	8	0 8	0	14	11	1 9
14	0	15	2	2 10	0	15	5	3 11	0	15	9	0 12	0	16	0	1 13
15	0	16	3	2 14	0	16	6	3 15	0	16	10	0 16	0	17	1	1 17
16	0	17	4	2 18	0	17	7	3 19	0	17	11	0 20	0	18	2	1 21
17	0	18	5	2 22	0	18	9	0 0	0	19	0	1 1	0	19	3	2 2
18	0	19	6	3 3	0	19	10	0 4	1	0	1	1 5	1	0	4	2 6
19	1	0	7	3 7	1	0	11	0 8	1	1	2	1 9	1	1	5	2 10
20	1	1	8	3 11	1	2	0	0 12	1	2	3	1 13	1	2	6	2 14
21	1	2	9	3 15	1	3	1	0 16	1	3	4	1 17	1	3	7	2 18
22	1	3	10	3 19	1	4	2	0 20	1	4	5	1 21	1	4	8	2 22

Excise for Common Brewers in *London*, at 1 *s.* 3 *d.* per Bar. *&c. Small Beer.*

Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
23	1	5	713	38	15	1403	76	5	2093	115	0
46	2	10	736	40	0	1426	77	10	2116	116	5
69	3	15	759	41	5	1449	78	15	2139	117	10
92	5	0	782	42	10	1472	80	0	2162	118	15
115	6	5	805	43	15	1495	81	5	2185	120	0
138	7	10	828	45	0	1518	82	10	2208	121	5
161	8	15	851	46	5	1541	83	15	2231	122	10
184	10	0	874	47	10	1564	85	0	2254	123	15
207	11	5	897	48	15	1587	86	5	2277	125	0
230	12	10	920	50	0	1610	87	10	2300	126	5
253	13	15	943	51	5	1633	90	0	2323	127	10
276	15	0	966	52	10	1656	91	5	2346	128	15
299	16	5	989	53	15	1679	92	10	2369	130	0
322	17	10	1012	55	0	1702	93	15	2392	131	5
345	18	15	1035	56	5	1725	95	0	2415	132	10
368	20	0	1058	57	10	1748	96	5	2438	133	15
391	21	5	1081	58	15	1771	97	10	2461	135	0
414	22	10	1104	60	0	1794	98	15	2484	136	5
437	23	15	1127	61	5	1817	100	0	2507	137	10
460	25	0	1150	62	10	1840	101	5	2530	138	15
483	26	5	1173	63	15	1863	102	10	5060	277	10
506	27	10	1196	65	0	1886	103	15	7590	416	5
529	28	15	1219	66	5	1909	105	0	10120	555	0
552	30	0	1242	67	10	1932	106	5	12650	693	15
575	31	5	1265	68	15	1955	107	10	15180	832	10
598	32	10	1288	70	0	1978	108	15	17710	971	5
621	33	15	1311	71	5	2001	110	0	20240	1110	0
644	35	0	1334	72	10	2024	111	5	22770	1248	15
667	36	5	1357	73	15	2047	112	10	25300	1387	10
690	37	10	1380	75	0	2070	113	15	27830	1526	5

The foregoing Tables (though very ſhort) will ſhew the exact Duty for any Number of Barrels, and their uſe will be rendered very plain and eaſie, by the Directions and Example following.

1. If the Number of Barrels of ſtrong or ſmall Beer given be leſs than 23, ſeek it in the firſt Column of the Table, and againſt it you have the exact Duty in Pounds, Shillings, Pence, Farthings, and 23th. parts of a Farthing. Thus by the firſt Table the Duty of 10 Barrels and an half will be found to be 2 *l.* 4 *s.* 5 *d.* 1 *f.* and $\frac{1}{2}\frac{8}{3}$ parts of a Farthing; for againſt 10 in the Column of Barrels, and under $\frac{1}{2}$ you have this ſum.

2. For any Number above 23, ſeek it in ſome of the Columns of Barrels in the Table, and if you cannot find the ſame Number, take the next leſs, and ſet down the ſum of Money againſt it, then Subtract the Number found from the Number given, the remainder (which will ever be leſs than 23) being found in the firſt Column of the Table gives you a ſum of Mony, which added to the former is the exact Duty required.

Example.

Example.

What Sum of Mony will the Duty of 1306 ½ *Barrels of ſtrong Beer amount unto?*

See the Work by the firſt Table.

	Bar.	*l.*	*s.*	*d.*	*f.*
Number given ——	1306 ½				
Next leſs *per* Table	1288 =	272 :	13 :	00 :	0 : 00
Remainder ——	18 ½ =	3 :	18 :	3 :	3 : 12
The exact Duty is ——		276 :	11 :	3 :	3 : 12

That is 276 *l.* 11 *s.* 3 *d.* 3 *f.* and ⅔ parts of a Farthing.

And ſo for any other Number of Barrels by this or any of the other Tables.

SECT: XI.

Shewing ſome Uſes of the

RULE

IN

ARITHMETICK

AND

GEOMETRY.

PARTICULARLY,

In the Reſolution of Queſtions relating to *Intereſt.*

The Rule of Fellowſhip, the proportion betwixt *Lines* and *Solids.*

The Meaſuring of *Superficies*, as *Board, Paving, Cieling, Flooring, Tyling*, &c. And alſo of *Solids*, as *Timber, Stone, Brick-work.*

SECT. XI.

Of Simple INTEREST.

Problem I.

If 100 *l. in* 1 *Year,* 12 *Months, or* 365 *days, doth gain* 6 *l. or Amount to* 106 *l. (at* 6 per Cent. per Ann.) *How much ſhall any other Sum gain, or amount unto, in any other Time at the ſame Rate?*

THIS Problem doth comprehend ſeveral Queſtions, the Chief whereof are the ſix next following.

Queſt. I.

To find the Intereſt of any Sum of Money in one Year at the Rate of 6 per Cent. per Ann.

FOR reſolving of this Queſtion, the Proportion is,

As 100 *l.* Principal, is to 6 its Intereſt in one year,

So is any other Sum to its proportional Intereſt.

Therefore,

Set 100 (in the middle) upon A, to 6 (the Intereſt thereof) upon B, then againſt any other Sum upon A, you have the Intereſt upon B. Thus the Lines being ſet as above directed, againſt 250 upon A, is 15 upon B; which is the Intereſt of 250 *l.* in one Year, and without moving the Rule I likewiſe find that if the Principal

l.		*l.*
1000		60
850		51
600		36
360	the Intereſt will be	21.6
130		7.8
85		5.1
50		3
20		1.2

Note, The parts of a Pound may be reduced to Shillings and Pence by the Rule, and when there be Shillings and Pence in the Principal, they muſt be reduced to a Decimal, by the directions given in page 46.

For Inſtance,

How much is the Intereſt of 56 l. 5 s. 6 d. in one Year?

The Shillings and Pence being reduced as above directed, the Sum will be 56.275 *l.*

As

Then ſay,

As 100 upon A, is to 6 upon B,
So is 56.275 upon A, to 3.3765 upon B.

The Anſwer is, 3.3765, which being reduced is 3 *l.* 7 *s.* 6 *d.* 1 *f.*

Queſt. II.

To find the Amount of any Sum of Money in one Year at 6 per Cent. per Ann.

THIS may be done by adding the Intereſt (found by the laſt) to the Principal; or by the Inſtrument the proportion is,

As 100 is to 106,
So is any other Sum to the Amount thereof in one Year,

Therefore,

Set 100 upon B, to 106 upon A, then againſt any other Sum upon B, you have the Amount upon A: Thus (the Rule being ſet as above directed) you will find that if the Principal

l.		*l.*
1000		1060
850		901
600		636
360	the Amount in	381.6
130	one year will be	137.8
85		90.1
50		53
20		21.2

Queſt.

Queſt. III.

To find the Intereſt that will be due for any Sum of Money, in any Time propounded, in Years, Months, or Days.

Rule.

FIND (by the firſt Queſt.) the Intereſt of the Sum given in one Year.

Then ſay,

As 1 year, 12 months, or 365 days, is to the Intereſt found.

So is the Time propounded (in years, months, or days,) to the Intereſt ſought.

I. For Years.

Example 1. *What is the Intereſt of* 250 *l. in* 6 *years, at* 6 per Cent. per Ann.

1. Set 100 upon A, to 6 upon B, then againſt 250 upon A, is 15 upon B, the Intereſt in one year.

2. *As* 1 upon A, is to 15 upon B,
So is 6 upon A, to 90 upon B, the Intereſt required.

II. For

II. For Months.

Example 2. *What is the Intereſt of* 250 *l. for three Months?*

The Intereſt of 250 *l.* in 12 months by the laſt Queſtion is 15 *l.*

Therefore ſay,

As 12 upon B, is to 15 upon A:
So is 3 upon B, to .375 upon A: That is, 3 *l.* 15 *s.* the Intereſt of 250 *l.* for three months.

III. For Days.

Example 3. *How much is the Intereſt of* 250 *l. for* 146 *days?*

Find (as before) the Intereſt of 250 for one year, which is 15 *l.* Then,

Set 365 upon A, to 15 upon B, and againſt 146 upon A, you have 6 upon B, that is 6 *l.* which is the Intereſt ſought.

In all theſe and ſuch like Caſes, the Intereſt thus found being added to the Principal gives the Amount in the time propoſed.

But the Intereſt, and alſo the Amount of any Sum of Money (eſpecially if it be great) may be more exactly found by the following Table.

In the firſt Column of which you have the ſeveral Rates *per Cent. per Ann.*

The ſecond Column ſhews the Intereſt of one Pound for one year at each reſpective Rate, found by dividing the ſeveral Rates by 100.

The

The third Column ſhews the Intereſt of one Pound for one month, and theſe are found by dividing the reſpective Rates by 1200.

The Numbers in the fourth Column ſhew the Intereſt of one Pound for one day, and are gotten by dividing the ſeveral Rates by 36500.

The uſe of this Table is, to find the Intereſt or Amount of any Sum of Money for any Number of years, months or days, at any Rate therein expreſt: for which obſerve this general Rule.

Multiply the Tabular number (which ſtands againſt the Rate propounded, and under the Title, year, month or day, according as the Queſtion is) by the number of years, months or days, for which you ſeek the Intereſt, the Product will be the Intereſt of one Pound for that time; which Multiplied by the Sum given, the leaſt Product will ſhew the Intereſt ſought.

Rates per Cent.	*Year.*	*Month.*	*Day.*
4	.04	.00333	.000109589
5	.05	.00416	.000136986
6	.06	.005	.000164383
7	.07	.00583	.000191781
8	.08	.00666	.000219178
9	.09	.0075	.000246575
10	.10	.00833	.000273972

Example:

Example: *What is the Interest of* 800 *l. for* 300 *days, at* 6 per Cent. per Ann.

Here the Question being for *days*, I seek in the Column under the Title *Day*, and against 6 the Rate propounded, and find .000164383, this Multiplied by 300 (the Number of Days) gives .0493149, which is the Interest of one Pound for 300 days; now this Multiplied by 800 (the Sum given) the Product will be 39.45192; that is, 39 *l.* 9 *s.* 1 *f.* and so much is the Interest of 800 *l* for 300 days, to which if you add the Principal (*viz.* 800) the Sum will be 839 *l.* 9 *s.* 1 *f.* the Amount of 800 *l.* in 30 days: Or (for the Amount) thus, find the first Product (*viz.*) the Interest of one Pound for the number of years, months or days propounded: this done, add an Unit thereto, the Sum will be the Amount of one Pound in that time; which Multiplied by the Sum whose Amount you seek, the last Product is the Answer.

Thus in the last Example the first Product was .0493149, to which add an Unit, the Sum is 1.0493149, (the Amount of one Pound in 300 days) this Multiplied by 800 the Sum given, the Product is 839.45192 the Amount as before.

This Question wrought by the Pen will be very exact, though the time be long and Sum great, but any ordinary Question may be resolved true enough by the Instrument.

Quest.

Queſt. IV.

Let there be ſuppoſed a Sum of Money preſently due, alſo the Sum it did amount unto in a time unknown, at a certain Rate per Cent. per Ann. *To find in what time it is ſo increaſed.*

The Rule.

SUbtract the Sum propoſed from the Amount, the the remainder is the Intereſt for the time ſought, which divided by the Principal, (or Sum due) then if the Quotient thus found be divided by the Intereſt of one Pound for one year, month or day, (found by the Table above) according to the tenure of the Queſtion, the laſt Quotient will be the time ſought in years, months or days, reſpectively.

Example.

If 252 *l. hath been forborn 'till with the Intereſt of* 6 per Cent. per Ann. *it is increaſed to be* 270 *l.* 18 *s.* (*or* 270.9.) *How many months hath it been forborn?*

The Amount	270.9
Sum firſt given	252.0
The Remainder	18.9

Is

Is the Interest for the time sought: This divided by 252, the Quotient is .075.

Now the time sought being Months, I find in the the former Table the Interest of one Pound for one month at 6 *per Cent.* (*viz.* .005) by which I divide the former Quotient (*viz.* 075;) the last Quotient is 15, the number of Months sought.

To work this by the Instrument, find the Interest as above, and then the proportion is,

1. *As* the Sum due upon B, is to 1 upon A,
So is the Interest thereof upon B, to a fourth term upon A.

2. *As* the Tabular Number (found as above) upon B, is to 1 upon A;
So is the fourth term upon B, to the time sought upon A.

Thus in the last Example, the Interest was found to be 18.9:

Therefore,

Set 252 upon B, to 1 upon A, then against 18.9 upon B, is .075 upon A.

Again,

Set .005 (the Interest of one Pound for one Month) upon B, to 1 upon A, then against .075 upon B, is 15 (the Months sought) upon A.

Quest.

Queſt. V.

A Sum of Money propoſed, and alſo the Intereſt due for the ſame for any time given, to find at what Rate of Intereſt it became due.

Rule.

MUltiply the Intereſt by 100, and divide the Product by the Principal Multiplied by the time given, this done the Quotient will ſhew the Rate of Intereſt *per Cent. per Ann.*

Note, We take a month for the $\frac{1}{12}$ part of a year, and in this Queſtion the time given, be it months or days muſt be reduced to Years and Decimal parts:

For which ſay,

As 12 to 1,
So is any number of months to the years and parts: And,
As 365 to 1,
So is any number of days to the years and parts: Now to the Queſtion,

Example: *If* 252 l. *being forborn* 15 *months did gain* 18 l. 18 s. (*or* 18.9.) *What Rate of Intereſt was it accompted at?*

The Intereſt (*viz.*) 18.9 Multiplied by 100 is 1890: And the Principal 252 Multiplied by the time,

me, *viz.* 1.25 is 315, by which divide the former roduct, (*viz.*) 1890, the Quotient will be 6, hich is the Rate of Intereſt ſought.

Queſt. VI.

A Sum of Money due at any time to come, at any Rate of Intereſt per Cent. per Ann. *to find the preſent worth.*

Rule.

DIvide the Sum given by the Amount of one Pound in the Time, and at the Rate pro poſed, the Quotient will be the preſent worth.

Example: *If* 270.9 l. *be due at the end of* 15 *months to come, What is it worth in ready Money diſcompting Intereſt at* 6 per Cent. per Ann.

The Amount of one Pound in any time may be found by the third Queſt. In 15 months it will be 1.075, by which divide 270.9, (the Sum propoſed) the Quotient will be 252 *l.* and ſo much preſent Money is worth 270.9 *l.* due at the end of 15 months to come, for 252 *l.* with the Intereſt thereof for 15 months will Amount to 270.9 as appears by the fourth Queſt.

Of

Of Annuities at Simple Intereſt.

Problem II.

To find the Amount of any Annuity for any time given, and at any Rate of Simple Intereſt.

A General Rule.

1. FInd the Intereſt of the Annuity for half the number of years, half years or quarters, that the Annuity is to continue.

2. Multiply the Intereſt thus found by the Number of years, half years or quarters, leſs by 1, and to this Product add the Sum of the Annual, half yearly, or quarterly payments, the Sum will be the true Amount of the Annuity.

Example: *What is the Amount of an Annuity of* 100 l. *in* 8 *Years?*

The Intereſt of 100 *l.* for half the time, *viz.* 4 years, is 24 *l.* this Multiplied by the number of years leſs by 1, (*viz.*) 7, the Product will be 168, to which add the Sum of the Annual payments, *viz.* 800, the Sum will be 968 *l.* the Amount ſought.

Problem

Problem III.

To find the preſent worth of any Annuity for any given time, at any Rate, accompting Simple Intereſt.

The Rule, or proportion is,

AS *the Amount of* 1 l. *in any time is to* 1 l. So *is the Amount of any Annuity, to the preſent worth.*

Therefore,

Divide the Amount of the Annuity by the Amount of one Pound in the time given, the Quotient is the preſent worth.

Example: *What is the preſent worth of an Annuity of* 100 l. *to continue* 8 *years?*

The Amount of 100 *l.* Annuity for 8 years, by the laſt Problem is found to be 968 *l.* and the Amount of one Pound in 8 years, will be found (by the ſecond Queſt.) to be 1.48.

Now if 968 (the Amount of the Annuity) be divided by 1.48, the Quotient is 654.054, and ſo much preſent Money is equal in value to an Annuity of 100 *l.* for 8 years, for if this 654.054 be put out at Intereſt, it will in 8 years amount to 968 *l.* as may be found by the Rule given in Queſt. 3.

Of

Of Compound Intereſt.

IN Compound Intereſt, the reſpective Amounts for each reſpective Year are ſo many Geometrical proportional Numbers. Thus, if 100 *l.* be put out to Intereſt at 6 *per Cent. per Ann.* there will be due at the end of the firſt year 106 *l.* which 106 is now made a Principal, and at the end of the ſecond year will Amount to 112.36, and this being made a Principal, will at the end of the third year be increaſed to be 119.1016, now theſe four Numbers, *viz.* 100, 106, 112.36, and 119.1016, are four Geometrical proportional Numbers:

For,

As 100 to 106, (for the firſt year)
So is 106 to 112.36. (for the ſecond year.)

Again,

As 106 to 112.36,
So is 112.36 to 119.1016, for the third year, *&c.*

Problem

Problem IV.

Any Sum of Money being propounded, to find the Amount thereof in any time, at any Rate of Compound Intereſt.

Example: *What is the Amount of* 100 l. *being forborn* 5 *years at* 6 per Cent. per Ann. *Compound Intereſt?*

1. SET 100 upon B, to 106 (the firſt years Amount) upon A; then againſt 106 upon B, you have 112.36 upon A, the Amount in two years.

In like manner.

2. Againſt 112.36 upon B, is 119.1016 upon A, the Amount in three years:

Again,

3. Againſt 119.1016 upon B, is 126.247 upon A, the Amount in four years.

Laſtly,

Againſt 126.247 upon B, you have 133.822 upon A, which is the Amount in 5 years, and the Anſwer to the Queſt.

Problem

Problem V.

A Sum of Money being due at any time to come, to find what it is worth in ready Money at any Rate of Intereſt?

Example: *What is the preſent worth of* 133.822 *l. due at the end of* 5 *years to come, at* 6 per Cent. per Ann. *Compound Intereſt?*

THis is but the Inverſe of the laſt Problem; Therefore if you ſee the Rule as is there directed, you may read the proportion back again: For the reſpective Amounts found upon A, in that Example will point you to the reſpective preſent worths at the end of each reſpective Year.

Thus,

Set 106 upon A, to 100 upon B; then againſt 133.822 upon A, is 126.247 upon B: The preſent worth of 100 *l.* at the end of the firſt year.

Alſo,

Againſt 126.247 upon A, you have 119.1016 upon B, the preſent worth at the end of the ſecond year.

Again,

Seek 119.1016 upon A, againſt it you have 112.36 upon B, the preſent worth at the end of the third year.

In

In like manner,

Againſt 112.36 upon A, is 106 upon B, the preſent worth at the end of the fourth year.

Laſtly,

Againſt 106 upon A, you have 100 upon B, which is the preſent worth of 133.822 *l.* at the end of five years which was required.

Problem VI.

A yearly Rent or Annuity being forborn a certain number of years, to find what the Arrearages thereof will amount unto at any Rate propounded.

Rule.

1. FInd a Sum (or Principal) whoſe Intereſt is equal to the Annuity or yearly Rent.
2. Find (by the fourth Problem) the Amount of that Sum in the time given: This done, Subtract the Principal firſt found from the Amount, the remainder will be the Anſwer.

Example: *A Rent or Annuity of* 9 l. per Ann. *being forborn* 12 *years, what will the Arrearages thereof amount unto accompting Intereſt upon Intereſt at* 6 per Cent?

To find a Principal whoſe Intereſt is 9 *l.*

Say,

As 6 to 100,
So is 9 to 150. the Principal ſought.

The Amount of 150 *l.* in 12 years, will be found by the fourth Problem to be 301.828, from which Subtract (the Principal firſt found, *viz.*) 150, the remainder will be 151.828, the Sum of the Arrearages ſought.

Problem VII.

A yearly Rent or Annuity being propounded, to find what it is worth in ready Money?

Rule.

FIrſt, find (by the laſt Problem) what the Arrearages thereof will amount unto at the end of the term propounded: Then (by the fifth Problem) find the preſent worth of thoſe Arrearages; the preſent worth thus found is the value of the Annuity or yearly Rent ſought.

Example: *Suppoſe a Rent of* 9 l. per Ann. *to continue* 12 *years, what ſum of Money may be given for this Rent, allowing the Buyer* 6 per Cent. *Compound Intereſt?*

BY the laſt Queſtion the Arrearages of a Rent of 9 *l. per Ann.* being forborn 12 years, were found to be 151.828 *l.*

And

And the preſent worth of 151.828, due at the end of 12 years to come will be found (by the fifth Problem) to be 75.443, and ſo much may be given for a Rent of 9 *l. per Ann.* to continue 12 years.

But ſuppoſe the term of the Rent or Annuity begins not 'till after the expiration of certain years to come, in ſuch caſe find what the Arrearages forborn for all that time are worth in ready Money, for that is the Anſwer.

For Inſtance.

Sudpoſe in the laſt Example, the Annuity of 9 *l.* for 12 years were not to being 'till after the expiration of 6 years: I here ſeek what the Arrearages (*viz.* 151.828) being forborn 18 years are worth in ready Money, and by the 5th. Problem I find it will be 53.185 *l.* which is the Anſwer: So a yearly Rent or Annuity of 9 *l. per Ann.* to begin 6 years hence, and continue 12 years, is worth but 53.185, in ready Money.

Problem VIII.

A Sum of Money being propounded, to find what Annuity (to continue any Number of Years, and at any Rate propoſed) that Sum will buy.

Rule.

TAke any Annuity at pleaſure, and find the value thereof in ready Money: This done the proportion is,

As the preſent worth found is to the Annuity taken,
As is the Sum given to the Annuity ſought.

Example: *What Annuity to continue 12 years will* 300 l. *Purchaſe, allowing the Buyer* 6 per Cent. *Compound Intereſt?*

By the 7th. Problem I find that 75.443 will Purchaſe 9 *l. per Ann.* to continue 12 years.

Therefore,

Set 75.443 upon B, to 9 upon A; then againſt 300 upon B, is 35.776 upon A, that is 35 *l.* 15 *s.* 6 *d.* and ſo much Annuity to continue 12 years is worth 300 *l.*

Problem IX.

There is a Free-hold Eſtate of the yearly value of 78 *l. What Sum of Money is this Eſtate worth, allowing the Buyer* 6 per Cent. per Ann. *Compound Intereſt for his Money?*

The Rule or Proportion is,

AS *the Annual, half yearly or quarterly Intereſt of* 1 *l. is to* 1 *l.*
So *is the Annual, half yearly or quarterly Rent to the Sum ſought.*

The

The Intereſt of 1 *l.* at 6 *per Cent.* Compound Intereſt:

For 1	year	is	.06
	half year		.02956
	quarter		.01467

Now to Anſwer the Queſtion.

Set .06 upon B to 1 upon A, then againſt 78 upon B is 1300 upon A, which is the value of the Eſtate propounded; and for the ſame reaſon I find without moving the Rule, that a Freehold Eſtate

of	90	Pounds	1500
	120	*per Ann.*	2000
	150	is worth	2500

Pounds.

But ſuppoſe the Rent of the Former Eſtate, (*viz.* 78 *l.*) were payable *Quarterly*, (*viz.* 19.5 *l. per* Quarter:) *What is it then worth*, allowing the Purchaſer 6 *per Cent.* as before?

Set .01467 (the Intereſt of 1 *l.* for a Quarter of a year) upon A, to 1 upon B; then againſt 19.5 (the Quarters Rent) upon A, you have 1329.244 upon B, that is 13.29 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* the Anſwer, which is 29 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* more than the former; that being accounted by yearly, this by Quarterly Payments.

Problem X.

A Sum of Money propounded, to find the Annual Rent of a Freehold Eſtate that ſuch a Sum will purchaſe, at a given Rate of Compound Intereſt.

THIS is the Inverſe of the laſt, and therefore the proportion is:

As 1 *to the Annual Intereſt,*
So *is the Sum propounded to the Annual Rent.*

Example: *Suppoſe I would lay out* 3650 l. *upon a Freehold Eſtate, ſo as to have* 6 per Cent. Comp. *Intereſt for my Money, What muſt the Annual Rent be?*

Set 1 upon A to .06 upon B; then againſt 3650 upon A, you have 219 upon B: The Anſwer is, 219 *l. per Ann.*

Problem

Problem XI.

The Annual Rent of a Freehold Estate being known, and also the Sum of Money which was paid for the same, to find what rate of Compound Interest the Purchaser shall have for his Money.

The Proportion is,

As the Sum given is to 1,
So is the Annual Rent to the rate of Interest.

Example: *There is* 1300 l. *given for a Freehold Estate, whose Annual Rent is* 78 l. *What rate of Interest has the Purchaser for his Money?*

Set 1300 upon A to 1 upon B; then against 78 upon A is .06 upon B: So the Purchaser shall have 6 *per Cent.* for his Money.

Lastly,

If it be demanded, *How many years Purchase any Freehold Estate is worth at any rate of Compound Interest?*

The Proportion is,

As the Rate of Interest is to 1,
So is 1 *to the Answer: Example, at* 5 per Cent.

Set .05 upon B to 1 upon A; then against 1 upon B, is 20 upon A; so at 5 *per Cent.* it is worth 20 years Purchase.

At 6 *per Cent.* It is worth but 16.66 years Purchaſe.

For,

As .06 upon B to 1 upon A,
So is 1 upon B to 16.66 upon A, that is 16 years 8 months and ſomewhat more.

The Converſe of this is *to find by the Number of years Purchaſe, what Rate of Intereſt the Buyer hath for his Money.*

Say,

As the Number of years Purchaſe is to 1.
So is 1 *to the rate of Intereſt.*

Example: *A Freehold Eſtate is bought at* 20 *years Purchaſe; What rate of Intereſt hath the Purchaſer for his Money?*

Set 20 upon A to 1 upon B, then againſt 1 upon A, is .05 upon B; that is, 5 *per Cent. per Ann.*

THE

THE RULE OF FELLOWSHIP.

Problem XII.

Five men C D E F *and* G *make a Bank, or Stock of Money in all* 360 l. *with which they Trade and gain* 40 l. *what part of this Gain muſt each man receive to ballance his Money put in.*

THE Inſtrument will reſolve this and ſuch like Queſtions by Inſpection.

The Proportion is,

As the whole Stock
is to the whole Gain:
So is each mans Particular Stock,
to his Particular Gain.

Therefore,

Set 360 the whole Stock upon A, to 40 the whole Gain upon B, then againſt each man's Stock upon A you have his Gain upon B. Thus againſt 108 upon A you have 12 upon B. So C who put in 108 *l.* ſhall have 12 *l.* of the Gain, and ſo of the reſt.

	Stock.	Gain.
C	108	12
D	81	9
E	63	7
F	72	8
G	36	4
	360	40

By which it appears that this Queſtion (which by the Pen would require five Operations) is reſolved by the Rule at one.

A Question of this Rule with Time.

Problem XIII.

Two Merchants join together, H *put in* 60 l. *for* 6 *Months,* F *put in* 80 l. *for* 8 *Months, and they gained* 50 l. *how much muſt each Merchant have of the Profit?*

The Proportion will be

(1.) *As* 1 *is to* 6 *(months)*
So is 60 l. *to* 360.

(2.) *As one is to* 8 *(months)*
So is 80 l. *to* 640.

Set 1 upon A to 6 upon B, then againſt 60 upon A is 360 upon B.

Again, Set 1 upon A to 8 upon B, the againſt 80 upon A, is 640 upon B, the ſum of theſe is 1000.

Now for each Merchants Share.

1. Set 1000 upon A to 50 upon B, then againſt 360 upon A is 18 upon B, that is 18 *l.* and ſo much is H Share.
2. Set 1000 upon A to 50 upon B, then againſt 640 upon A is 32 upon B, which is the Share belonging to F.

So

So that $\left\{\begin{matrix}H\\F\end{matrix}\right\}$ must have $\left\{\begin{matrix}18\ l.\\32\ l.\end{matrix}\right.$

The whole Gain 50 *l.*

The Use of the Rule in Military Affairs.

Problem XIV.

Any number of Men being propoſed, to place them in Battalia, ſo that there may be as many in Rank as in File, and ſuppoſing them to be imbattelled at Order (which is 3 Foot diſtance in Rank and File) to find how many Square foot of Ground they will take up?

Example.

LET the Number of Men propoſed be 3600. ſet the Lines C and D even at the end. Now foraſmuch as the Number given doth conſiſt of an even Number of Figures, ſeek the Number, *viz.* 3600 in the ſecond Radius on the Line C, and againſt it on the Line D, you will find 60, and ſo many Men muſt be in Rank, and as many in File:

Now to find how many Square Foot of Ground they will occupy:

Set 1 upon A to 3 upon B, then againſt 60 upon A is 180 upon B.

Then ſet the Lines C and D even, and find this Number, *viz.* 180 on the firſt Radius on the Line

D,

D, and againſt it you have 32400 upon C and ſo many Square-foot of Ground will theſe Soldiers occupy, being at their Order of three Foot.

Problem XV.

Any Number of Men being propoſed to place them in Battalia, in ſuch order that there may be twice as many in Rank as in File, and that they may ſtand at cloſe Order, *which is* 1 ½ *Foot.*

Example.

LET the Number given be 1800, Set the Lines D and C even, and count half the Number given, *viz.* 900 upon the Line C, and againſt it you have 30 upon D which is the depth of the File, and then there muſt be 60 in Rank, which is twice 30.

Now for the Ground that theſe will occupy being at Cloſe Order.

Set 1 upon A to 1.5 upon B, then againſt 30 upon A is 45 the Side of the Ground, ——and without moving the Lines ſeek 60 upon the Line A, and againſt it you have 90 upon B, which is the Length of the Ground.

Now for the Area.

Set 1 upon A to 45 upon B, then againſt 90 upon A is 4050, and ſo many ſquare Foot of Ground will theſe 1800 Men Occupy at Cloſe Order.

Problem

Problem XVI.

Any Number of Men being proposed to be put in Battalia and a certain Number named to be either in Rank or File, to find the other Number?

Example.

LET it be required to place 1200 Men in Battalia, so that there shall be 15 in File, how many must there be in Rank; or how many Files must there be?

This is resolved by the Lines A and B. And the Proportion is

As 15, the depth in File is to 1.

So is 1200 the Number proposed to 80 the Number of Men in Rank.

Problem

Problem XVII.

Any Number of Soldiers being given together with their distance in Rank and File, to order them into a Square Battail of Ground?

Example.

LET the Number of Soldiers given be 5400, their distance in File 6 Foot, and in Rank 4 Foot.

Set 6 upon the Line A to 4 upon the Line B, and then against 5400 upon A is 3600 upon B.

Then set the Lines D and C even, and seek 3600 in the second Radius upon C, against it you have 60 upon D, the Number of Men to be placed in File.

Now for the Number of Men in Rank, say,

As 60 is to 1: So is 5400 to 90.

So there shall be 60 in File and 90 in Rank.

Now if you Multiply 60, the Number of Men in File, by 6 their distance, the Product will be 360, the Side of your Ground.

In like manner, if you Multiply 90, the Number of Men in Rank, by 4 their distance, the Product will be 360 as above, which shews your Ground is exactly Square, and is therefore a proof of the Work.

Problem

Problem XVIII.

If 1000 *Soldiers may be Lodged or Quartered in a ſquare Piece of Ground, whoſe ſide is* 400 *Foot: how many Foot long muſt the ſide of a Square be that the Ground included may Lodge* 4000?

The Proportions will be

1. *As* 1 to 400 (the ſide of the Square which will Lodge 1000 Soldiers.)
So is 400 to 160000.

2. *As* 1000 is to 4000,
So is 160000 to 640000.

Now (Setting the Lines D and C even) find this 640000 in the ſecond Radius on the Line C, and againſt it you have 800 upon D, and ſo much muſt the ſide of a Square be, to Lodge 4000 Soldiers with the ſame convenience that 1000 were Lodged in a Square, whoſe ſide was 400 Foot.

Triplicate Proportion *is such a Proportion as is between* Lines *and* Solids, *or between* Solids *and* Lines, *of which I shall give some Examples in the following* Problems.

Problem XIX.

There is a Bullet whose Diameter is 4 *Inches, and the weight* 9 *pounds, what shall another Bullet of the same Metal weigh, whose Diameter is* 8 *Inches?*

SET 4 (the Diameter of the Bullet proposed) upon the single Line, to 9 the weight on the Line E, then against 8 upon the single Line is 72, which is the weight of the Bullet whose Diameter is 8 Inches.

And as the Lines now stand you may find the weight of all Bullets of the same Metal; for against the Diameter upon the single Line you have the weight upon the Line E.

Thus if the Diameter of a Bullet

Be { 3, 5, 6, 7 } Inches, the weight will be { 3.79, 17.57, 30.37, 48.23 } Pounds.

Having the weight of two Bullets of the same Metal, and the Diameter of one of them to find the Diameter of the other.

This

This is but the Converſe of the laſt, and therefore without moving the Lines, find the weight on the Line E, and againſt it you have the Diameter on the ſingle Line.

Thus if a Bullet

Weigh	20	Pounds, the	5.23	Inches.
	30	Diameter	5.97	
	40	will be	6.67	

Problem XX.

If a Braſs Gun, whoſe Diameter is 8 Inches weighs 640 pounds, what ſhall another Gun weigh (of the ſame Shape and Metal) whoſe Diameter is 6 Inches.

SET 8 upon the ſingle Line to 640 on the Line E, then againſt 6 upon the ſingle Line is 270 on the Line E, which is the weight of the Piece whoſe Diameter is 6 Inches.

Problem

Problem XXI.

There is a Conical Tun which contains 620 *Gallons, The Diameter at the bottom is* 80 *Inches, the Diameter at the top* 71.2 *Inches, and the depth* 30 *Inches, it is desired to make another Tun in the same Shape with the former, that may contain* 800 *Gallons. To find the Diameter of this Tun at the Bottom and Top, and also the depth?*

For the Diameter at the Bottom.

SET 620 (the Content of the first Tun) upon the Line E, to 80 (the Bottom Diameter) on the single Line: then against 800 (the Content of the second Tun) upon E is 87 which is the Diameter sought.

2. *For the Diameter at the Top.*

Set 620 upon E to 71.2 upon the single Line, then against 800 upon E is 77.4 the Diameter sought.

3. *For the Depth.*

Set 620 upon E to 30 upon the single Line, then against 800 upon E is 32.63 the depth required.

So the Diameters and depth of this Tun are as followeth,

Bottom

Bottom	Diameter	87	Inches.
Top		77.4	
	Depth ——	32.63	

But the first of these being found as above directed, the other may be discover'd at one operation by the Lines A and B. Thus:

Set 80 upon A to 87 upon B, then against 71.2 upon A is 77.4 upon B, and against 30 upon A is 32.63 upon B, which also shews, that the Diameters and Depth of this Tun are in proportion to the Diameters and Depth of the Tun first proposed. And the like may be done for any Casks, of which I need not give any Examples.

Problem XXII.

There is a Ship of 60 Tun Burthen, which is 44 Foot by the Keel, and it is required to make another Ship of 360 Tun Burthen, of the same Shape with the first, How many Foot long must the Keel of the second Ship be?

SET 60 upon E to 44 upon the single Line, then against 360 upon E is 80 on the single Line which is the Length of the Keel required.

Now suppose in the first Ship the middle Beam be 17.6 Foot, and the depth of the Hold 7.76 Foot.

To

To find these Dimensions of the second Ship.

Set 44 upon A to 80 upon B, then against 17.6 upon A is 32 upon B. which is the length of the Beam. And without moving the Lines, seek 7.76 upon A, and against it you have 14.1 upon B, which is the depth of the Hold. So the Dimensions and burthen of the two Ships are as followeth.

	1 Ship	2 Ship	
Length of the Keel	44	80	Foot.
Length of the Beam	17.6	32	Foot.
Depth of the Hold	7.76	14.1	
Burthen	60	360	Tun.

To find the Burthen of any Ship, the Common Rule is,

Multiply the length of the Keel by the length of the Midship-Beam, then Multiply that Product by the depth of the Hold; then divide this last Product by 100 *the Quotient gives you the Tuns. But when there is no allowance for Guns, divide the last Product by* 95.

Thus in the second Ship, 80 Multiplied by 32 is 2560, this Multiplied by 14.1 is 36096. and this divided by 100 is 360; that is, 360 Tun which is the Burthen of this Ship, allowing for Guns, but if there be no allowance for Guns, divide by 95, and in this case her Burthen will be found to be near 380 Tun.

Proper-

Proportions in a Circle.

IT was shewn in Page 37 of this Book, that if the *Diameter* of a Circle be 1, the *Circumference* will be 3.141592, and the *Area* .785398, the Square-Root of .785398 is .886221 and this is the side of a Square which is equal to the Area of a Circle whose Diameter is 1. And the Square-Root of .5, *viz.* .707106 is the side of the greatest Square that can be inscribed in a Circle whose Diameter is 1? So that if the Diameter of a Circle be 1,

The	Circumference	3.141592
	Square equal	.886221
	Sq. Inscribed	.707106
	Area ——	.785398

Note further that if the Circumference of a Circle be 1,

The	Diameter	will	.318310
	Sq. equal		.282092
	Sq. Inscribed	be	.225079
	Area ——		.079577

If the Area of a Circle be 1.

The	Diameter ——	is	1.128378,
	Circumference		3.54491,

It

If the Circumference of a Circle be 4,

The { Diameter / Area —— } is 1.27324.

If the Diameter of a Circle be 4,

The { Circumference / Area —— } is 12.56636.

In all these Cases the *Area* is Square Inches, the other Terms (being Lines,) are Lineal Inches. These things Premised, I shall here add some useful Questions which may be resolved either by the Pen or Instrument.

1. *The Diameter of a Circle given, to find the Circumference.*

As 1 is to 3.141592, Or
As .31831 is to 1 :
So is the Diameter to the Circumference.

2. *The Circumference given to find the Diameter.*

As 1 is to .31831, Or
As 3.141592 is to 1 :
So is the Circumference to the Diameter.

3. *The Diameter given, to find the side of a Square equal.*

As 1 is to .886221, Or.
As 1.128378 is to 1 :

So

So is the Diameter to the side of a Square equal.

4. *The side of a Square given, to find the Diameter of a Circle whose Area is equal to the Square.*

As 1 is to 1.128378, Or
As .886221 is to 1:
So is the side of the Square to the Diameter sought.

5. *The Circumference given, to find the side of a Square equal to the Circle.*

As 1 is to .282092, Or
As 3.54492 is to 1:
So is the Circumference to the side of the Square required.

6. *The side of a Square given, to find the Circumference of a Circle equal to the Square.*

As 1 is to 3.54492, Or
As .282092 is to 1:
So is the side of the Square to the Circumference sought.

7. *The Diameter of a Circle given, to find the side of the greatest Square Inscribed.*

As 1 is to .707106, Or
As 1.41421 is to 1:
So is the Diameter to the side required.

8. *The side of a Square given, to find the Diameter of a Circle that shall Circumscribe the Square.*

As 1. is to 1.41421, Or
As .707106 is to 1:
So is the side of the Square to the Diameter sought.

9. *The Circumference of a Circle given, to find the side of the Square Inscribed.*

As 1. is to .225079, Or
As 4.4429 is to 1:
So is the Circumference to the side required.

10. *The side of a Square given, to find the Circumference of a Circle that shall Circumscribe the Square.*

As 1. is to 4.4429, Or
As .225079 is to 1:
So is the side of the Square to the Circumference sought.

11. *The Diameter of a Circle given, to find the Area.*

As 1. is to .78539, Or
As 1.27324 is to 1:
So is the Square of the Diameter to the Area required.

12. *the Area of a Circle given, to find the Diameter.*

As 1 is to 1.27324, Or
As .78539 is to 1:
So is the Area to the Square of the Diameter, whose Square Root is the Diameter sought.

13. *The Circumference of a Circle given, to find the Area.*

As 1 is to .079578, Or
As 12.56636 is to 1:
So is the Square of the Circumference to the Area sought.

14. *The Area of a Circle given, to find the Circumference.*

As 1 is to 12.56636, Or
As .079578 is to 1:
So is the Area to the Square of the Circumference, whose Square Root is the Circumference required.

The

The Use of the Instrument in Measuring of Timber and Stone.

Of Round TIMBER.

Problem XXIII.

The Circumference and Length of any Round Tree, being given (in Foot measure) to find the Content.

Rule.

THE Circumference of that Circle whose Area is 1 is 3.544.

Therefore,

As 3.544 upon D. is to the Length upon C.
So is the Circumference upon D, to the Content upon C.

Example.

Suppoſe the Length of a Tree be 20 Foot, and the Circumference 4 Foot, what is the Content?

Set 3.544 upon D, to 20 (the Length) upon C, then againſt 4 the Circumference upon D, you have 25.46 upon C, that is, 25 Feet and almoſt an half and ſo much is the Content.

Problem XXIV.

The Circumference of a Round Tree being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Rule given in the laſt Problem will ſerve here, if inſtead of 3.544 you make uſe of 42.53 (which is found by Multiplying 3.544 by 12.)

Example.

The Circumference is 36 Inches, and the Length 10 Feet, What is the Content?

Set 42.53 upon D, to 10 upon C, and againſt 36 upon D, is 7.16 upon C, which is the Content ſought.

Problem

Problem XXV.

The Circumference of any Round Tree being given, to find how much in Length it will take to make a Solid Foot.

FIrſt in Foot-Meaſure, the Proportion is,

As the Circumference upon D, is to 1 upon C,
So is 3.544 upon D, to the Length ſought upon C.

Example.

If the Circumference of a Tree be 3 Feet, How many Feet in length will it take to make a Solid Foot?

Set 3 upon D, to 1 upon C, and againſt 3.544 upon D, is 1.396 upon C, that is, 1 Foot and .396 parts. Now ſo many times as this is contained in the Length of the Tree, ſo many Feet does the Tree contain.

Secondly, In Inch Meaſure, if the Circumference be given in Inches:

Say,

As *the Circumference upon* D, *is to* 12 *upon* C.
So *is* 42.53 *upon* D, *to the length in Inches* (*upon* C) *to make a Foot.*

Example.

The Circumference of a Tree is 48 Inches, How many Inches in Length will take to make a Foot.

Set 48 upon D, to 12 upon C, and againſt 42.53 upon D, is 9.42 upon C, that is, 9 Inches and .42 parts, the Length required.

Now ſuppoſing this Tree were 20 Foot long, the Content will be found to be 25.48 that is 25 Foot and almoſt a half, for 20 Foot is 240 Inches and 9.42 Inches is contain'd 25.48 times in 240.

Note The common way uſed for the Meaſuring of Round Timber is to Girt the Tree about in the middle with a Line, and to take a fourth part of the Circumference for the ſide of a Square equal to the Circles, but this is falſe though moſt Men uſe it, for according to this Suppoſition it will take 12 Inches in length of the Tree above mentioned to make a Foot, and ſo the whole Tree would contain but 20 Foot, whereas the true Content is 25.48 Foot as was ſhewn above.

Note, In all that hath been ſaid of Round Timber, we ſuppoſe the Tree to be equally thick from end to end. But ſeeing moſt Trees are leſs towards the Top than at the Root, the Circumference muſt be taken in the middle, and if the ends differ much it will be more exact if you divide the Length into two equal parts, and take the Circumference in the middle of each, and ſo find the Content as if they were two Trees, Theſe Contents added together give the Content of the whole.

For Inſtance,

There is a Tree 12 Foot long, the Circumference at one end is 6 Foot, and at the other end 3 Foot, the Sum of theſe is 9, the half is 4.5 the Circumference in the middle, by which (and the length) the Content will be found (by Problem XXIII.) to be 19.36 Foot.

But if this Tree be divided into two parts, ſo as each may be 6 Foot long, the Circumference in the middle of the greater will be 5.25, by which the Content will be found to be 13.19 Foot.

And the Circumference in the middle of the leſſer piece will be 3.75, and the Content of this will be 6.72, which added to the Content of the other, *viz.* 13.19 gives 20 Foot, the Content of the whole Tree: Whereas the Content found by one Circumference taken in the middle of the whole Tree is but 19.36.

But in moſt Caſes it will be ſufficient to take the Circumference in the middle of the Tree, and ſo find the Content as in Problem XXIII. aforegoing.

Obſerve alſo, in Taper Trees when it is ſaid ſo much in length will make a Foot, it muſt be underſtood in that place only where the Circumference was taken, for it will not take ſo much in length of the greater end of a Tree to make a Foot as it will of the leſſer.

Yet, if by the Circumference taken in the middle you find the length of a Foot, that length ſo found will ſhew the whole Content of the Tree, as in Problem XXV.

Problem XXVI.

The Circumference of any Round Tree being given either in Feet or in Inches, to find how many Solid Feet are contain'd in one Foot of the length.

IF the Circumference be given in Feet, Set 3.544 upon D, to 1 upon C; then againſt the Circumference upon D, is the Content upon C.

Example.

Let the Circumference of a Round Tree be 4 Foot, How many Solid Feet are contain'd in one Foot of the length of this Tree? Anſwer 1.275:

For,

As 3.544 upon D, is to 1 upon C,
So is 4 upon D, to 1.275 upon C.

If the Circumference be given in Inches, make uſe of 42.53, inſtead of 3.544, and then the Proportion will hold as above.

Problem

Problem XXVII.

The Diameter and Length of any Round Tree, being given in Foot-measure to find the Content.

Rule.

THE Diameter of a Circle whose Area is 1, is 1.128:

Therefore say,

As 1.128 upon D, is to the length upon C,
So is the Diameter upon D, to the Content upon C.

Example.

The Diameter is 2 Foot, and the length 20 Foot What is the Content?

Set 1.128 upon D, to 20 upon C, and against 2 upon D, is 62.83 upon C, the Content required.

Problem XXVIII.

The Diameter given in Inches and the Length in Feet, to find the Content in Feet.

THIS differs not from the laſt, if inſtead of 1.128 you make uſe of 13.53 (which is found by Multiplying 1.128 by 12.)

Example.

The Diameter is 24 Inches, and the length 20 Feet, what is the Content?

Set 13.53 upon D, to 20 upon C, and againſt 24 upon D, is 62.83 upon C, the Content as before.

Problem XXIX.

The Diameter of any Round Tree being given, to find how much in length it will take to make a Foot.

1. *In Foot-Meaſure: The Proportion is,*

AS *the Diameter upon* D, *is to* 1 *upon* C,
So *is* 1.128 *upon* D, *to the length ſought upon* C.

Example.

Example.

The Diameter is 2 Foot:

Therefore,

Set 2 upon D, to 1 upon C, and against 1.128 upon D, is .318 upon C, and ſo many parts of a Foot in length, it will take to make a Foot.

2. *In Inch-Meaſure: The Proportion is,*

As the Diameter upon D, is to 12 upon C,
So is 13.53 upon D, to the length ſought upon C.

Example.

The Diameter is 24 Inches:

Therefore,

Set 24 upon D, to 12 upon C, and againſt 13.53 upon D, is 3.81 upon C, that is, 3 Inches and .81 parts, the length ſought.

Of

Of Square Timber.

Problem XXX.

The Length and Side of any piece of Square Timber, being given in Foot-Meaſure to find the Content.

The Proportion is,

As 1 *upon* D, *is to the length upon* C:
So *is the ſide upon* D, *to the Content upon* C.

Example.

There is a piece of Timber exactly Square, each ſide at either end being 1.5 Foot, and the length 20 Foot, How many Solid Feet doth this piece contain?

Set 1 upon D, to 20 upon C, and againſt 1.5 upon D, is 45 upon C, which is the Content ſought.

Problem

Problem XXXI.

The ſide of any Square Solid being given in Inches, and the length in Feet, to find the Content in Feet.

THE Proportion in the laſt Problem will hold here, if inſtead of 1 you make uſe of 12 for the firſt Term.

Example.

Let the ſide be 18 Inches, and the length 20 Foot, Whas is the Content in Feet? Anſwer 45 : *For*,

As 12 upon D, is to 20 the length upon C:
So is 18 the ſide upon D, to 45 the Content upon C.

When the ends are not exactly Square, (which doth often happen) take the breadth and thickneſs at the end in Foot and Parts, then Multiply the breadth by the Thickneſs, and this Product by the length, the laſt Product will be the Content.

If the breadth and thickneſs be given in Inches, and the length in Feet, then by the Lines A and B on the Rule.

The Proportion is,

1. As 1 *upon* B, *to the breadth upon* A:
So *is the thickneſs upon* B, *to a fourth Number, viz:* (the Area of the end in Square Inches.)

2. *As* 144 *upon* B, *to the fourth Number upon* A:
So *is the length upon* B, *to the Content ſought.*

But if any ſuch Solid be greater at one end than at the other, the exact way to find the Content, is to find the Area of each Baſe (in Foot-meaſure) then find a Geometrical Mean betwixt the ſaid Area's, then if the ſum of theſe three be Multiplied by $\frac{1}{3}$ part of the length, the Product is the Content.

The Uſe of the Rule in the menſuration of Brick-work.

BRick-work is commonly meaſured by the Rod-Square, which is 16.5 Feet in length, and as much in breadth, and by conſequence each Square Rod doth contain 272.25 Square Foot, for 16.5 Multiplied by 16.5 is 272.25.

Note alſo, all Brick-work muſt be reduced to the Standard-meaſure of one Brick and an half thick.

Problem

Problem XXXII.

The length and height of any Brick-Wall being given in Feet, to find the Content in Square Rods at any thickneſs.

The Rule.

MUltiply the height by the length, and divide the Product by 272.25, the Quotient will be the true Content if the Wall be juſt 1 ½ Brick thick:

If the thickneſs be more or leſs: *Then ſay,*

As ½ to the Number of half Bricks in the thickneſs of the Wall:

So is the Quotient above found, to the true Content ſought.

Or having the length and height in Feet, you may find the Content more readily on the Rule by this Proportion.

As 272.25 upon B, *to the height upon* A:

So is the length upon B, *to a fourth Number upon* A, which is the true Content if the Wall be 1 ½ Brick thick: If the thickneſs be more or leſs, *Say,*

As 3 upon B, *to the ½ Bricks thick upon* A:

So is the fourth Number above found upon B, *to the Content required.*

Example.

Example.

There is a Wall 16 Foot high, and 97 Foot long: How many Square Rod doth this Wall contain at 1 ½ Brick thick? Answer 5 Rod and 7 tenth parts: *For*,

As 272.25 upon B, to 16 the height upon A.
So is 97 the Length upon B, to 5.7 upon A.

Now if the Wall above mention'd were 2 ½ Bricks thick, it would Contain 9 Rod and an half: *For*,

As 3 upon B, to 5 (the ½ Bricks in 2 ½) upon A:
So is 5.7 (the Content at 1 ½ Brick thick) upon B, to 9.5 the Content ſought upon A.

But if a Wall of 16 Foot high, and 97 Foot long were but 1 Brick thick, the Content would be but 3 Rod and 8 tenth parts: *For*,

As 3 upon B, to 2 (the ½ Bricks in 1) upon A:
So is 5.7 (the Content at 1 ½ thick) upon B, to 3.8 the Content upon A.

Now that (in all theſe Caſes) you may find the Content at one operation: I ſhall here lay down certain fixt Numbers for any thickneſs hereafter expreſt,

viz.

viz. at	1	Bricks thick	408.37
	1½		272.25
	2		204.19
	2½		163.35
	3		136.12
	3½		116.68
	4		102.20
	4½		90.75
	5		81.75

By these Numbers the Content of any Wall, at any thickness here mention'd, may be found either by the Pen or Rule, and the Proportion is,

As *the Number proper to the thickness is to the height:*
So *is the length to the Content.*

Example.

There is a Wall 4 ½ Bricks thick, 7 Foot high, and 85 Foot long: How many Square Rod doth this Wall contain?

The proper Number for 4 ½ Bricks thick is 90.75.

Therefore,

Set 90.75 upon A, to 7 the height upon B; then against 85 the length upon A, is 6.555 upon B; that is, 6 ½ Rod and something more.

Note, The Decimal parts of a Rod may be reduced into Feet by this proportion.

As

As 1.00 *upon* A, *to* 272.25 *upon* B,
So *is any Decimal upon* A, *to the Number of Feet upon* B.

And thus you may find the Decimal in the laſt Example, *viz.* .555 is equal to 151 Foot.

Alſo,

¼ or .25	of a	68	
½ or .50	Rod	136	Feet.
¼ or .75	is	204	

Problem XXXIII.

The height of any Wall being given in Feet, to find how many Feet in length it will take to make a Square Rod at any thickneſs above mention'd.

The Rule.

SET the height upon A, to the Number proper for the thickneſs upon B, then againſt 1 upon A, is the length ſought upon B.

Example.

There is a Wall 1 ½ Brick thick, and 8 Foot high: How many Feet in length will it take to make a Square Rod? Anſwer, 34. *For,*

The

The proper Number for 1 ½ thick is 272.25.

Therefore,

As 8 (the height) upon A, to 272.25 upon B:
So is 1 upon A, to 34 upon B.

The Uſe of the Rule in the menſuration of Superficies, as Board, Paving, Cieling, Wainſcoting, Flooring, Tyling, &c.

NOte, In all Square or oblong Superficies the Area or Content may be found by this one proportion:

As one upon A, *to the breadth upon* B:
So *is the length upon* A, *to the Content upon* B.

This proportion will hold in Triangular Superficies alſo, if you take ½ the Perpendicular for the breadth, and the ſide upon which it falls for the length: And in all Caſes the Area thus found is,

Square { Inches / Feet / Yards } if the ſides were given in { Inches. / Feet. / Yards. }

But in many Caſes the ſides are given in one kind of Meaſure, and the Area required in another: I ſhall therefore give ſome Examples of each.

I. For

I. *For Board.*

IF the length and breadth be both given in Inches, and the Content ſought in Feet.

Say,

As 144 *to the breadth:*
So is the length to the Content.

Example.

A Board is 14 Inches broad, and 156 Inches long: How many Square Feet doth it contain? Anſwer 15.16.

For,

As 144 upon A, to 14 the breadth upon B:
So is 156 the length upon A, to 15.16 the Content upon B.

If the breadth be given in Inches, and the length in Feet:

Say,

As 12 *to the breadth:*
So is the length to the Content.

Example.

A Board is 14 Inches broad, and 13 Foot long: What is the Content in Feet? Anſwer 15.16

For,

For,

As 12 upon B, to 14 the breadth upon A:
So is 13 the length upon B, to 15.16 the Content upon A.

II. *Of Paving.*

PAviers Work is measured by the Yard Square, which contains 9 Square Feet: The most natural way, is to measure the sides with a Yard divided into 100 equal parts, and then Multiply the breadth by the length, the Product will be the Content in Square Yards: But if the sides be given in Feet.

Say,

As 9 to the Breadth:
So is the length to the Content.

Example.

There is a Court Paved with Stone, the breadth is 64 Foot, and the length 95 Foot: What is the Content in Square Yards? Answer, 67.55

For,

As 9 upon A, to 64 the breadth upon B:
So is 95 the length upon A, to 67.55 the Content upon B.

III. *Of Cieling.*

Cieling, and alſo *Wainſcoting* are meaſured by the Yard Square, and therefore what is ſaid in the Example above may be underſtood here.

IV. *Of Flooring and Tyling.*

THeſe are both meaſured by the Square which contains 100 Square Foot, the ſide being 10 Foot: Therefore if the length and breadth be given in Feet, Multiply one by the other, and cut off two Figures in the Product, the Remainder ſhews the Content in Squares, and the Figures cut off are the parts of a Square, or odd Feet.

Or by the Rule ſay,

As 100 *to the breadth in Feet:*
So *is the length in Feet to the Content in Squares.*

Example.

There is a Floor 66 Foot broad and 75 Foot long: How many Squares doth it contain? Anſwer 49.5.

For,

As 100 upon A, to 66 the breadth upon B:
So is 75 the length upon A, to 49.5 the Content upon B.

The like for Tyling.

Laſtly,

Lastly,

The breadth of any Superficies being given in Feet or Yards, to find how many in length (of either) it will take to make a Foot or Yard Square.

The Proportion is,

As the breadth to 1 :
So *is* 1 *to the length sought.*

Example.

A Board is 1.5 Foot broad: how much in length will it take to make a Foot Square? Answer .667.

For,

As 1.5 (the breadth) upon A, is to 1 upon B.
So is 1 upon A, to .667 upon B.

So that if one side of a Piece of Board be 1.5 Foot, the other side must be .667 of a Foot to make a Square Foot; for .667 Multiplied by 1.5 is 1.

In Inch measure the Proportion is

As the Breadth in Inches is to 12,
So *is* 12
To the length in Inches to make a Foot.

And

And.

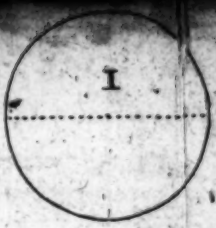
As the Breadth in Inches is to 36,
So is 36
To the Length in Inches to make a Yard.

Example.

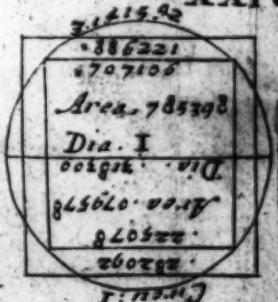
A Board is 18 Inches Broad, how many Inches in length will make a Foot.

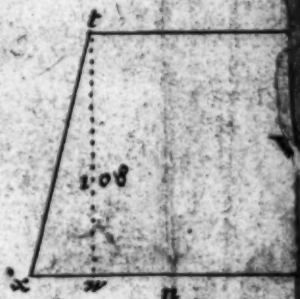
Set 18 (the Breadth) upon A to 12 upon B, then against 12 upon A is 8 upon B, and so many Inches in length will make a Foot of this Broad; for 18 Multiplied by 8 gives 144 the Square Inches in a Square Foot.

APPEN-


I

XXIV


3.1415.92
.886221
.707106
Area. 785398
Dia. I
Dia. .318309
Area .079578
.225078
.282092
Circu: I


t
1.08
x
w

n
130
b

BRITISH MUSEUM
30
60
NOV

APPENDIX.

Of Conical Sections.

IN the Doctrine of Conicks there are these two things (very different) to be separately considered. First, What Figures or Curve-Lines do arise from the Section of a Cone by a Plain. And secondly, What is the nature of such Figures or Curve-Lines, considered without respect had to such supposed Original, by cutting a Cone.

See Dr. *Wallis de Con. Sect.* and Treatise of *Algebra*, page 291.

Definition.

1. *A Cone is a Figure made when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the place from whence it first moved: the Axis of a Cone is that fixed Side about which the Triangle is moved: the Base of the Cone is a Circle, which is described by the containing Side that is moved about.*

Of the ſeveral Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis; the Section will be a Triangle, as S V B, Figure 1.

2. If a Cone be cut by a Plain Parallel to the Baſe, the Section will be a Circle, whoſe Diameter let be S D B. Figure 1, 2. and 3.

3. If a Cone be cut by a Plain that is Parallel to the ſide of the Triangle (S V) the Crooked Section o o A o o is a Parabola; in which P A is the Diameter, or Axis, and D o an Ordinately applied Line. *vide* Figure 1.

4. If a Cone be cut by a Plain, that will alſo cut one of the ſides of the Triangle produced in E, (Figure 2.) the Crooked Section o o o A o o o is an Hyperbola; in which H A is the intercepted Diameter, A E the Tranſverſe Diameter, and D o an Ordinately applied Line.

5. If a Cone be cut by a Plain through both ſides of the Triangle under the Vertix V, the Crooked Section o o o A o o o (Figure 3,) is an Ellipſis; in which a A is the Diameter, and D o an Ordinately applied Line.

Theſe are all the Sections that can be made by cutting a Cone by a Plain. I ſhall Treat only of the three laſt: and of theſe I ſhall not ſay all that might be ſaid; but ſo much, and in ſuch method as I conceive moſt agreeable to my preſent deſign; which is only

only to explain the Nature and Original of ſome few Solids, which are frequently mentioned in Books (and do ſometime occur in the Practice) of Gauging.

Of the Parabola.

IN the Parabola Figure 1. Let the Cone be cut by a Plain (S D B) Parallel to the Circular Baſe (S P B) it will appear.

1. S D is equal to S P (for A P is Parallel to S V, and S B is Parallel to S B.)

2. S P Multiplied by P B is equal to P o ſquared, *ergo* S D multiplied by D B is equal to D o ſquared.

3. As A P to P o ſquared, So is A D to D o ſquared.

4. *Ergo*, As A P to A D, So is P o ſquared to D o ſquared, which is the general property of a Parabola.

5. Divide the Square of P o by A P, or the Square of D o by A D, in either caſe the Quotient will be the right ſide, for which put L, according to this all the powers of the Ordinates (D o) are proportioned; for whereſoever D be taken, it will be L × A D = D o q, that is, the right ſide multiplied by A D is equal to the Square of D o.

Hence may be found out a way (having the Axis A P, and the right ſide L.)

To deſcribe a Parabola in Plano.

1. THrough the Axis A P (Figure 4.) and at Right Angles to it, let be drawn a competent number (the more the better) of Parallel Lines, as S S, S S, *&c*, then whereſoever D be taken, it will

be A D × L = D o q, that is, D o is a mean Proportional between A D and the Right ſide, this gives the Points o (as many as you pleaſe) through which the Crooked Line A o o o being drawn on both ſides, you have the Parabola deſired.

A ſecond way without knowing the Parameter is this.

To the Lines A P and P o draw the Line A o, completing the Triangle A P o, then,

A P : P o :: A D : D e. and

P o × D e = D o q.

A third way.

Draw A N Parallel to P o, divide P o and A N into any Number of equal parts, and divide A P equally to the Square of thoſe parts, as here we divide A N into 6 parts, and A P into 36, then make the firſt n c = 1, the ſecond n c = 4, the third = 9, the fourth = 16, the fifth = 25 of theſe 36 parts, ſo ſhall the Points o, be all in the Parabola, and throught theſe Points it may be drawn.

Note, The Triangle o A o is $\frac{3}{4}$ of the Parabola o o A o o, and the Parabola o o A o o is $\frac{2}{3}$ of the Parallelogram N o o V, theſe properties agree to any Parabola, as is demonſtrated by Dr. *Wallis* in his *Mechanica*, Part 2. Chap 5. where 'tis alſo ſhewn that if the Semi-Parabola P o o A, be turned about upon the Line P A, it will deſcribe the Parabolick Conoid o o A o o, whoſe magnitude is equal to half the Circumſcribing Cylinder o N V o, that is, a Parabolick Conoid to a Cylinder of the ſame Baſe and Altitude is, as 1 to 2, and hence the three following Problems are eaſily reſolved.

Problem

Problem I. Figure 4.

Given o P o *the Diameter of the Baſe, and* A P *the Altitude of a Parabolick Conoid, to find the Solid Content.*

MUltiply the Area of the Baſe by half the Altitude, the Product is the Content ſought, and this will hold whereſoever the Baſe be taken, for if the Parabolick Conoid whoſe Baſe is o P o, and Altitude P A, be cut with a Plain Parallel to the Baſe, (as ſuppoſe o H o) the part cut off, *viz.* o A o H is a true Parabolick Conoid, having all the affections of the other (of which it was a part) and therefore its Content is equal to half the Circumſcribing Cylinder O n r o, and hence the Content of the Fruſtum o H o o P o may be alſo obtained; for the Parabolick Conoid whoſe Baſe is o P o, leſs the Parabolick Conoid, whoſe Baſe is o H o, is equal the Fruſtum o H o o P o, and thus the Content of the Fruſtum o o L o o, equal the Fruſtum o B G o, may be found, and theſe too Fruſtums o o L o o, and o G B o, abutting upon the common Baſe o P o, are what ſeveral Writers would have to repreſent a Cask, but 'tis obvious, from the Nature and Conſtruction of the Figure, thaa no Cask can be in this form: I ſhall not therefore examine whether the Rules they give for finding the Content of ſuch (ſuppoſed) Cask be true or falſe; for let that be as it will, they are of little or no uſe in the buſineſs of Cask Gauging.

But if a Tun shall be in the form of the Fru ſtum of a Parabolick Conoid, the Content of th whole, and also upon every Inch of such Tun ma be readily found by what hath been already sai and shall be further declared in the two nex Problems.

Problem II. Figure 4.

Let B o P o G *represent a Tun in the form of the Fruſtum of a Parabolick Conoid, whoſe Content is sought.*

TO the Area of the greater Base o P o add the Area of the lesser Base B L G, then Multiply their sum by half the depth or Altitude (*viz.* ½ P L) the Product is the Content. Or,

Get the Area of the Circle in the middle of the depth (*viz.* y y) this Multiplied by P L gives the Content of the whole Tun.

Problem

Problem III. Figure 4.

There is a Tun in the form of the Fruſtum of a Parabolick Conoid, o H o *the Diameter of the leſſer Baſe is* 20 *Inches,* o P o *the Diameter of the greater Baſe* 40, *and* H P *the depth* 30, *to find the Content of this Tun upon every Inch.*

BY (o P o) the Diameter of the greater Baſe, divide the Square of (o H o) the Diameter of the leſſer Baſe, the Quotient is equal to Z H e, ſubtract this from the Diameter of the greater Baſe, and by the remainder divide the Rectangle of the greater Diameter o P o, and Tuns depth H P, the Quotient equal to A P the Axis of the Parabolick Conoid, by this Axis divide the Square of the Diameter of the greater Baſe, the Quotient reduced into Ale-Gallons (by dividing it by 359.05) is the difference between the Content of the firſt and ſecond Inches of the Tun; by this difference multiply the Axis leſs .5, it gives the Content of the firſt Inch next the greater Baſe; from which ſubtract the ſaid difference, there will remain the Content of the ſecond Inch, and from this ſubtract as above, there reſt the third Inch, and ſo of the reſt till you come to the leſſer Baſe.

Example.

The Square of o H o is 400, this divided by o P o *viz.* 40, quotes 10, equal to Z H e, this taken from o P o 40, there reſts 30, by this divide 30 the Rectangle of o P o 40, into H P 30, (that is 1200)

the Quote is 40, equal A P the Axis of the Parabolick Conoid; the Square of o P o 40, is 1600, this divided by the Axis 40, gives 40, and this divided by 359.05 Quotes .1114 of an Ale-Gallon, and such is the difference between the Content of the first and second Inches of this Tun. Lastly, The Axis A P 40 less .5 is 39.5, this multiplied by the difference last found, gives 4.4004 the Content of the first Inch of the Tun next the greater Base, from which the aboveſaid difference (*viz.* .1114) being Subtracted, there rests 4.2890 the Content of the second Inch, and so by a continued Subtraction of the said difference, the Content upon every Inch is found as in this following Table, these added together make 83,5530 the whole Content of the Tun; and if we find the whole Content of this Tun by the two Problems aforegoing, it will be the same with this, *viz.* 83.553

Diam.		Area's.
o P o =	40	4.4562
o H o =	20	1.1140
Sum is		5.5702
$\frac{1}{8}$ H P =		15
		278510
		55702
Content =		83.5530 Ale-Gallons.

A Table shewing how many Ale-Gallons the foregoing Tun will contain upon every Inch of its depth.

Tuns Depth.	Content upon every Inch.	Com. differ.
1	4.4004	
2	4.2890	.1114
3	4.1776	.1114
4	4.0662	.1114
5	3.9548	.1114
6	3.8434	.1114
7	3.7320	.1114
8	3.6206	.1114
9	3.5092	.1114
10	3.3978	.1114
11	3.2864	.1114
12	3.1750	.1114
13	3.0636	.1114
14	2.9522	.1114
15	2.8408	.1114
16	2.7294	.1114
17	2.6180	.1114
18	2.5066	.1114
19	2.3952	.1114
20	2.2838	.1114
21	2.1724	.1114
22	2.0610	.1114
23	1.9496	.1114
24	1.8382	.1114
25	1.7268	.1114
26	1.6154	.1114
27	1.5040	.1114
28	1.3926	.1114
29	1.2812	.1114
30	1.1698	
	83.5530	

Note, from this Table may be Collected another, that ſhall ſhew the Number of Gallons contained in this Tun at any depth, as hath been already done in Sect. VIII. of the foregoing Tract.

Of a Parabolick Spindle.

IF the Parabola G A H (Fig. 5.) ſhall be turned round about upon the right Line G H, it will deſcribe a Solid as G A H B G, which Solid is called a Parabolick Spindle.

Now every Parabolick Spindle is to its Circumſcribing Cylinder, as 8 to 15. (*vid. Dr. Wallis, Mechanica* Part. 2. Chap. 5. Probl. 12.)

Therefore the Parabolick Spindle G A H B G, is $\frac{8}{15}$ of the Cylinder I K M L.

Problem IV. Figure 4.

To Calculate the Diameters, and thereby to find the Content of the middle Fruſtum of a Parabolick Spindle, whoſe greateſt Diameter A B *is* 32 *Inches, the leſſer* C D 24, *and the length* P S 40, *(or of the whole Spindle* G A H B D.)

1. DIvide the three given Diameters in the middle in the Points P Q S: through theſe Points draw the ſtraight Line Z Z, make b E Parallel to Q S, then ſhall A b be 4, A Q 16, and b e 20; and becauſe it is a Parabola it will be

A b : A Q : : b E q : Q H q that is,

4 : 16 : : 400 : 1600

the Square Root of 1600 is 40 equal to Q H.

2. Now to find a Diameter in the middle of every Inch of G Q (or P Q) through the Point A draw I K Parallel to G H and equal to it; let I A

and

and G H, be each divided into half Inches, which here will be 80, the Square of this is 6400, by which Divide A Q 16, the Quotient will be .0025, this Multiplied by the Squares of all the odd Numbers, *viz.* 1. 3. 5. 7, *&c.* to 79, the several Products give the length of the line n e, which taken from the Line A Q leaves the Lines e o the Semidiameters of the Parabolick Spindle, one in the middle of every Inch of G Q. But to contract this Work, we need but the three first Diameters, the rest are obtained by Subtracting the Common difference, as will appear by the following Operation.

.0025	1	is	.0025	each of
Multiplied by	9	is	.0225	each of
	25		.0625	

these taken from A Q 16 the three remainders will

be	15.9975	these Multiplied by 2 gives	31.995	
	15.9775		31.955	
	15.9375		31.875	

the three Diameters, one in the middle of every Inch next to A B, to find the Common Difference, take the second Diameter from the first there rests .040; take the third from the second, the remainder is .080, take .040 from .080 there rests .040, the second difference, which is the same throughout, and by this Common Difference the rest of the Diameters are thus found. Take the first difference (*viz.* .040) from the first Diameter, the remainder is the second Diameter; add the second Difference to the first Difference, and Subtract their sum from the second Diameter, the remainder will be the third Diameter, and so of the rest, as in the following Table; in the first

first Column of which you have the true Diameters, one in the middle of every Inch of G Q. In the second Column are the first Differences, in the third, the second Difference, which is the same throughout, in the last Column you have the Area's of the several Circles in Ale-Gallons, or (which is the same thing) the Content upon every Inch of the half Spindle A G D B A, the sum of these is 60.817, this doubled is 121.634, and such is the Content of this Parabolick Spindle: and for confirmation of this Rule, it may be observed, that the Content here found (viz. 121.634) is 8 fifteen parts of the Cylinder I K M L, whose Diameter A B is 32, and length G H 80 Inches, for the Area of 32 is 2.851, this Multiplied by 80, is 228.08, the Content of the Cylinder, this Divided by 15, gives 15.205, and this Multiplied by 8, is 121.64, which agrees with what we have cited above out of Dr. *Wallis*.

Diameters

	Diameters	First Diff.	Sec. Diff.	Areas or Cont.
1	31.955	80		2.851
2	31.955	80	40	2.844
3	31.875	120	40	2.829
4	31.755	160	40	2.808
5	31.595	200	40	2.780
6	31.395	240	40	2.747
7	31.155	280	40	2.705
8	30.875	320	40	2.656
9	30.555	360	40	2.610
10	30.195	400	40	2.540
11	29.795	440	40	2.472
12	29.355	480	40	2.400
13	28.875	520	40	2.322
14	28.355	560	40	2.224
15	27.795	600	40	2.150
16	27.195	640	40	2.060
17	26.555	680	40	1.963
18	25.875	720	40	1.862
19	25.155	760	40	1.761
20	24.395	800	40	1.658
21	23.595	840	40	1.549
22	22.755	880	40	1.442
23	21.875	920	40	1.332
24	20.955	960	40	1.222
25	19.995	1000	40	1.112
26	18.995	1040	40	1.002
27	17.955	1080	40	0.895
28	16.875	1120	40	0.791
29	15.755	1160	40	0.691
30	14.595	1200	40	0.592
31	13.395	1240	40	0.500
32	12.155	1280	40	0.412
33	10.875	1320	40	0.328
34	9.555	1360	40	0.251
35	8.195	1400	40	0.187
36	6.795	1440	40	0.128
37	5.355	1480	40	0.078
38	3.875	1520	40	0.041
39	2.355	1560	40	0.017
40	0.795			0.002

The Area

The ſum of the firſt 20 Inches is 48.24', and ſo much is the Content of the Fruſtum CABD, this doubled gives 96.49, the Content of CAEFBD, the middle Fruſtum of the Parabolick Spindle, which was required.

Note, The middle Fruſtum CAEFBD, may repreſent a Cask.

And if a Cask be in this Form, and the Diameters and length as above ſuppoſed, its Content is 96.49 Ale-Gallons.

Whereas by Mr. *Jordaine's* Rule in his *Duodecimal Arithmetick*, page 291. it would be but 94.108.

And by Mr. *Hunt's* Rule, in his *Gagers Magazine*, page 257, it will be but 93.877.

And (which is ſtill further from the truth) it would be but 87.936, by Mr. *Hunt's* Rule, in his *Practical Gaging Epitomized*, page 72. which is more than 8 Gallons too little.

Of the Hyperbola.

IN Fig. 2. the Cone SVB is cut by a Plain HA, which alſo cuts one ſide of the Triangle produced in E, therefore the Crooked Section oooAooo is an Hyperbola, EA is the Tranſverſe Diameter, and AH the intercepted Diameter. Now if the Cone be cut by another plain Parallel to the Baſe, this Section will be a Circle, whoſe Diameter is SDB: and SD × DB = Doq, therefore Do is an Ordinately applied Line, and if Do be a Line Ordinately applied, it will be

AH × EH : AD × ED :: Hoq : Doq

In

In words thus,

As A H Multiplied by E H, is to A D Multiplied by E D.

So is H o Squared, to D o Squared.

And if we make it

As A D Multiplied by E D, is to D o Squared.

So A E to L, the fourth term L will be the Right ſide, with the help of which the Ordinates may be all found by this Proportion.

As the Tranſverſe Diameter is to the Right ſide.

So is the Rectangle under the Intercepted Diameter, and the Sum of the Tranſverſe Diameter and Intercepted Diameter, to the Square of the Ordinate ſought.

Therefore if the Tranſverſe Diameter A E, Intercepted Diameter A D (in Fig. VI.) and the Right ſide be given the Hyperbola may eaſily be deſcribed in Plano, for the Ordinates D o may be all found by the proportion above, which gives the points o o o, through which the Hyperbola muſt be drawn.

Of an Hyperbolick Conoid.

IF the Semi-Hyperbola A S S S D be turned round about upon the Line A D, it will deſcribe the Hyperbolick Conoid S S A o o, and the Content of this Hyperbolick Conoid may be eaſily found; for if we put the Tranſverſe Diameter = t, the Intercepted Diameter = d, the magnitude of the Hyperbolick Conoid = A, and the Magnitude of a Cylinder of the ſame Baſe and Altitude = B, it will be

$3t + 2d : 6t + 6d :: A : B.$ therefore
$6t + 6d : 3t + 2d :: B : A.$

In

In words thus,

1. To 6 times the Tranſverſe Diameter A E, add 6 times the Intercepted Diameter A D, the Sum is your Diviſor.

2. To 3 times the Tranſverſe Diameter, add twice the Intercepted Diameter, by this Sum Multiply the Content of the Cylinder N V O S, the Product is your Dividend; which Divided by the Diviſor above found, the Quotient will be the Content of the Hyperbolick Conoid, whoſe Baſe S D o, is the Baſe of the Cylinder N V O S, and Altitude A D equal to S N the Altitude of the Cylinder, and this will hold whereſoever the Baſe be taken; for ſuppoſe the firſt S D o next the Vertex A, be taken for the Baſe of the Hyperbolick Conoid S A o, (and of the Cylinder S n u o, the Content of this Hyperbolick Conoid may be found by this Rule, and ſo the Content of any Fruſtum, S D o o D S may be likewiſe found, for the little Hyperbolick Conoid S A o, taken from the Hyperbolick Conoid S S S A o o o, there remains the Fruſtum S D o o D S, and thus the Content of the whole. and alſo the Content of any Fruſtum, and conſequently of every Inch of the Hyperbolick Conoid may be readily found, as will appear in the following Example.

Example. Fig. VI.

Let A E the Tranſverſe Diameter, and A D the Intercepted Diameter be each 32 Inches, and S D o the Diameter of the Baſe 45.255 Inches: to find the Content of the Hyperbolick Conoid in Ale-Gallons.

1. Six times A E is 192, and 6 times A D is 192, the Sum of theſe is 384. for the firſt term of the proportion.

2. Three

2. Three times A E is 96, and twice A D is 64, the Sum of these is 160 for the second term.

3. The Diameter of the Base S D o is 45.255 (or the Square Root of 2048;) the Area of this Circle is 5.7039 Ale-Gallons, this multiplied by the Altitude A D or N S, *viz.* 32 gives 182.524 the Content of the Cylinder N S o V, and this is the third term of the Proportion, which will stand thus,

384 : 160 :: 182.524 :

Multiply the third by the second, and Divide the Product by the first, the Quotient or fourth term will be 76.05, and such is the Content of the Hyperbolick Conoid in Ale-Gallons.

To find the Content of any Frustum S D o, o D S, *of the Hyperbolick Conoid, Fig. VI.*

BY the directions above the Content of the little Hyperbolick Conoid, whose Altitude I n is 4, and so the Diameter of its Base 12 Inches, will be found to be .7724 parts of a Gallon, this taken from the Content of the Hyperbolick Conoid above found, *viz.* 76.05. the remainder is 75.2776, the Content of the Frustum S D o, o D S.

And thus the Content of every Inch may be found: for let the first A D be 1 Inch, the second 2 Inches, and the third 3 *&c.* by the Rule above given

the	1 Hyperb.	1 Inch	will be	.04549
	2 Conoid whose	2 Inches	found	.18567
	3 Altitude is	3 Inches	to be	.42611

The first of these, *viz.* .04549 taken from the second,

ſecond, the remainder is .14018 the Content of the ſecond Inch S D o, o D r, in like manner the ſecond Hyperbolick Conoid, *viz.* .18567 taken from the third, there will remain .24044 the Content of the third Inch r D o, o D S.

Having thus got the Content of the three firſt Inches, a Table may be made by Addition for the whole Hyperbolick Conoid thus.

Set down the three firſt Inches as in the ſecond Column of this Table, then take the firſt Inch from the ſecond, the difference is .09469, ſet this in the third Column, take the ſecond Inch from the third, the difference is .10026, ſet this in the third Column under the other, then take .09469 from .10026 the difference is .00557, and this will be the ſame throughout, as in the fourth Column.

Now this .00557 added to .10026 is .10583 (the third Number in the third Column) and this added to the third Inch gives .34627 the Content of the fourth Inch, and ſo by a continual Addition the Table is made.

Now ſuppoſe the Hyperbolick Conoid, whoſe Altitude is 32 Inches, were divided into ſo many parts of equal thickneſs, by Plains S D o Parallel to the Baſe, the Content of every of theſe parts are expreſs'd by the Numbers in the ſecond Column of this Table, and the Sum of all theſe is 76.04912, which is the true Content of the Hyperbolick Conoid within leſs than $\frac{1}{1020}$ of a Gallon.

Inches

Inches.	Content of every Inch of the Hyp. Conoid.	First Differ.	Second Differ.
1	.04547	.09469	
2	.14018	.10026	.00557
3	.24044	.10583	.00557
4	.34627	.11140	.00557
5	.45767	.11697	.00557
6	.57464	.12254	.00557
7	.69718	.12811	.00557
8	.82529	.13368	.00557
9	.95897	.13925	.00557
10	1.09822	.14482	.00557
11	1.24304	.15039	.00557
12	1.39343	.15596	.00557
13	1.54939	.16153	.00557
14	1.71092	.16710	.00557
15	1.87802	.17267	.00557
16	2.05069	.17824	.00557
17	2.22893	.18381	.00557
18	2.41274	.18938	.00557
19	2.60212	.19495	.00557
20	2.79707	.20052	.00557
21	2.99759	.20607	.00557
22	3.20368	.21166	.00557
23	3.41534	.21723	.00557
24	3.63257	.22280	.00557
25	3.85537	.22837	.00557
26	4.08374	.23394	.00557
27	4.31768	.23951	.00557
28	4.55719	.24508	.00557
29	4.80227	.25065	.00557
30	5.05292	.25622	.00557
31	5.30914	.26179	.00557
32	5.57093		

76.04912 = Con. of the Hyp. Con.

To

To Describe an Hyperbola in Plano Geometrically.

FRom the middle of the Right Line R R (fig. VII.) and at Right Angles to it; draw the Line W A B, then divide W R into any Number of equal parts, and draw the Lines r o b Parallel to W B, this done set one point of the Compasses in A, and extend the other to r, where keep it fast whilst the other point is applied to the Line r b, it will give the point o, and thus if all the Lines A r be transferred to the Lines r b, from r towards b, they will give the points o o o through which the Crooked Line O o o A o O, being drawn you have the Hyperbola required, in which W A is half the Transverse Diameter, and A B the Intercepted Diameter (*vid.* Dr. *Wallis's Mechanica*, Part 2. pag. 553.)

Of the Hyperbolick Spindle.

IF the Hyperbola O o o A o O (Fig. VII.) be turned round about upon the Right Line O B O it will describe the Solid O o A o O E D E, which Solid is called a Hyperbolick Spindle.

Now supposing the Plain O A D, to be made up of innumerable Right Lines Parallel to A D, whereof one is o e, the quantity of these Lines may be found as shall be shewn below, and although in Mathematical strictness, Lines cannot fill a Plain by reason they have no breadth, yet the quantity of the Lines o e added together (supposing those Lines innumerable) is so near the true Area of the Plain

O A D

O A D that the difference will be leſs than any aſſignable quantity.

In like manner ſuppoſing the Semi-Hyperbolick Spindle O A D to be made up of innumerable Circles, whoſe Diameters are the Lines o e, the Content of all theſe Circles added together, is near equal to the Content of the Semi-Hyperbolick Spindle O A D, and if theſe Circles be infinitely many, the difference will be infinitely ſmall, or leſs than any aſſignable. (*vid.* Dr. *Wallis's Treatiſe of Algebra*, page 286.) The length of the Lines o b may be thus found.

To the Square of W A add the Square of W r, the Sum is the Square of r o, (per 47. 1 Eucl) and the Square-Root of r o, that is, the Line r o it ſelf, taken from the Line W B, the remainder is the Line o b, this doubled is the Diameter or Line o e, and this will hold whereſoever the point r be taken, and ſo innumerable Diameters o e may be found on both ſides A D towards o, and conſequently the Circles filling the Hyperbolick Spindle, and therefore the Spindle it ſelf being made up of theſe its Content is alſo known.

But for our preſent purpoſe it will be ſufficient to find a Diameter in the middle of every Inch of O B, and then we ſhall have ſo many Circular Plains of one Inch thick (which here we take as Cylinders) the Content of theſe added together, is the Content of the Semi-Hyperbolick Spindle as near as is requiſite in ordinary Practice, I ſhall indeavour to illuſtrate what hath been ſaid, by ſhewing the Content of the middle Fruſtum E O A o E D in Ale-Gallons.

Example

Example.

Let W A be 48 Inches, A B 16, W B 64, the length of the Fruſtum b b = E E 40, the greateſt Diameter A D 32, and the leſſer E o 24 Inches.

Suppoſe the firſt W r = .5 the Square of W A 48 is 2304, to this add the Square of W r, *viz.* .25 the Sum is 2304.25.

Let the ſecond W r be 1.5 the Square of this *viz.* 2.25 added to 2304 is 2306.25. and ſo of the reſt as in the following Table.

In the firſt Column of which you have the Squares of W r, W r; r being taken in the middle of every Inch of the firſt 20 Inches from W towards R.

In the ſecond Column are the Sum of the Squares of W A and W r, that is, the Squares of A r or r o.

In the third Column is the Square Roots of the Numbers in the ſecond, that is, the length of the Lines r o, each of theſe Numbers taken from W B 64, leaves the Lines o b, which doubled are the Diameters o e, as in the fourth Column.

In the laſt Column are the Areas of theſe Diameters in Ale-Gallons, or the Content upon every Inch of the Portion or Fruſtum A D E o, the Sum of theſe is 48.062, which double is 96.124, the Content of the middle Fruſtum o E D E o A which was required.

Whereas by Mr. *Hunt's* Rule in his *Gagers Magazine*, page 257, the Content of this Fruſtum would be but 90.31 Ale-Gallons, which is about 6 Gallons too little.

A

A Table shewing the Content of every Inch of the middle Frustum of the Hyperbolick Spindle E o A o E D (Fig. VII.) in Ale-Gallons.

Squares of *Wr.*	Sum of the Squares of *WA.* and *Wr.*	Square-Roots of the Squar of *Ar* or *ro*, or the length of the lines *ro.*	Diameters *oo.*	Area's of the Circles *oo*,
.25	2304.25	48.003	31.994	2.850
2.25	2306.25	48.023	31.954	2.836
6.25	2310.25	48.065	31.870	2.820
12.25	2316.25	48.127	31.746	2.800
20.25	2324.25	48.210	31.580	2.772
30.25	2334.25	48.314	31.372	2.738
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.836	2.649
72.25	2376.25	48.747	30.506	2.591
90.25	2394.25	48.931	30.138	2.528
110.25	2414.25	49.135	29.730	2.460
132.25	2436.25	49.358	29.284	2.386
156.25	2460.25	49.601	28.798	2.308
182.25	2486.25	49.862	28.276	2.225
210.25	2514.25	50.142	27.716	2.138
240.25	2544.25	50.440	27.120	2.047
272.25	2576.24	50.757	26.486	1.952
306.25	2610.25	51.091	25.818	1.854
342.25	2646.25	51.442	25.116	1.755
380.25	2684.25	51.810	24.380	1.653

Sum is ——— 48.062
This doubled is ——— 96.124
The Content of Frustum E o A o E D.

Of

Of an Ellipſis.

IF the Cone S e B d V (Fig. 3.) be cut by a Plain through both the ſides of the Triangle under the Vertex V, the Crooked Section o o A o o is an Ellipſis, in which A a is the Tranſverſe Diameter or Axes, and A D a Diameter intercepted.

Let the Cone be cut by another Plain (S B) Parallel to the Baſe S e B d, it will be S D x D B = D o q (per 35. 3. *Eucl.*) that is, S D Multiplied by D B, is equal to the Square of D o, and this will hold whereſoever D be taken, and give the quantity of the Lines D o, which are called Ordinately applied Lines, and the Lines o D o, which are ſuppoſed to cut the Line A a, at Right-Angles are called the Conjugate Diameters of the Ellipſis.

To Deſcribe an Ellipſis in Plano, the Tranſverſe Diameter A E, *and the Conjugate Diameter* H T, *or the Ordinate* S T, *being given.*

Figure VIII.

DRaw ſtraight Lines Parallel to H T, as b b t, o B P, &c. then,

As A S Multiplied by E S, is to A B Multiplied by E B,

So is the Square of S T to the Square of B P.

This will hold whereſoever B P be taken, and ſo a competent number of Points P, being found a Curved Line, A P P E drawn through thoſe Points, and

and continued the same on the other side (as H g N h A) you have the Ellipsis required.

Or thus,

Let it be,

As A S Multiplied by E S is to the Square of S T,

So is A E to E L, (E L is called the *Latus Rectum* or Right Side) draw the Line E L Parallel to S T, and joyn A L, then

A B Multiplied by B r, will be equal to the Square of B p.

Or thus,

As A E to E L :

So is A B Multiplied by B E to the Square of B P.

Another way may be this,

With the Radius A S describe the Quadrant A d D, and let the Lines T H, t h, be continued till they touch the Circle in D and d,

It will be S d q — S b q = b d q (per 47. 1. Eucl.) then,

S D q : S H q :: b d q : b h q. this gives the point h, through which the Ellipsis must be drawn.

Or Mechanically thus,

Take in the Compasses, the extent S A or S E, set one Foot in H, and bring the other to the Line A E, it will give the points n n, these points are called the *Nodes* of the Ellipsis.

Now it is evident from the conſtruction of the Figure, that the diſtance from the Center S to the Circumference of the Circle (d or D) and from thence to the Center again, is equal to the Diameter of the Circle; and alſo the diſtance from one Node to the Ellipſis (in H) and from thence to the other Node, is equal to twice the Radius S D, or to the Line A E: therefore,

As by tying a thread (twice the length of the Radius) about a fixed point in the Center, drawing another point in the extremity about, it will deſcribe a Circle: So by tying a thread about three Pins fixed in the points n n and H, and carrying the point H about in the extremity of the thread (the other two points remaining fixed) it will deſcribe the Ellipſis, for as S D S is equal to S d S: So n H n is equal to n g n.

Of a Spheroid.

IF the Semi-Ellipſis A t T P E be turned round about the Line A E, it will deſcribe the ſolid Figure A t P E g H h, which Solid is called a Spheroid.

1. The Content of every Spheroid is equal $\frac{2}{3}$ of the Content of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Altitude the ſame; therefore the Area of the greateſt Circle of a Spheroid Multiplied by $\frac{2}{3}$ of the length gives the Content.

2. If the Spheroid be cut in the middle of A S by a Plain z z parallel to H T, the Content of the Fruſtum Z A Z, to the Content of a Cylinder whoſe Baſe is z z, and Altitude z A, is as 5 to 9, therefore

therefore the Area of the Base Z z Multiplied by $\frac{1}{3}$ of x A gives the Content of the Frustum Z A Z.

3. Suppose the Spheroid cut as before, the lower Frustum Z H g E P P Z is equal to a Cylinder, whose Base is z z, and Altitude x E. (*vid.* Dr. *Wallis's* Treatise of Algebra, page 312.) these three proportions do likewise hold in a Sphere.

To Inch a Spheroid. Figure VIII.

LET H T the greatest Conjugate Diameter be 20 Inches, and A E the Transverse Diameter or Axis be 40 Inches, to find how many Ale-Gallons this Spheroid will contain upon every Inch.

Suppose Lines drawn parallel to H T, through every Inch of A S, these Lines may represent so many Diameters of the Spheroid, and the Squares of these may be found by the proportions above mentioned, for finding the Ordinates (B P) of the Ellipsis, in this Example the Square of the first Diameter at one Inch from H T, will be found to be 399, the Square of the second 396, and the third 391; and by these the Content of the 3 first Inches (which are sufficient for our present purpose) may be thus found.

1. To the Square of the first Diameter, viz. 399, add 800 (the double Square of H T) the sum is 1199, this divided by 1077 Quotes 1.1132 the Content of the first Inch in Ale-Gallons.

2. To 396 the Square of the second Diameter, add 800, the sum is 1196, this Multiplied by 2 gives 2392, Divide by 1077 the Quotient will be 2.2209, the Content of the two first Inches.

3. To 991 the Square of the third Diameter add 800, the Sum, viz. 1191, Multiplied by 3 is 3573, Divide as above, the Quote will be 3.3175, the Content of the three first Inches. Having got these three numbers, take the first from the second, the remainder is 1.1077, the Content of the second Inch, in like manner take the second from the third, the remainder will be 1.0966 the Content of the third Inch, So the Content of

The	First.	Inch is	1.1132
	Second		1.1077
	Third		1.0966

As in the second Column of the following Table. Take the second Inch from the first, the difference is .0055, take the third Inch from the second, the difference is .0111, set these two differences in the third Column: then take .0055 from .0111, the difference is 0056, which place in the fourth Column, this will be the same throughout: being thus prepared add .0056 to .0111, the Sum is .0167, take this from the third Inch, viz. 1.0966, the remainder is 1.0799, the Content of the fourth Inch, and so of the rest as in the Table.

Inch

Inch.	Content upon every Inch.	First Diffe.	Second Differ.
1	1.1132		
		.0955	
2	1.1077		.0056
		.0911	
3	1.0966		.0056
		.0867	
4	1.0799		.0056
		.0823	
5	1.0576		.0056
		.0279	
6	1.0297		.0056
		.0335	
7	0.9962		.0056
		.0391	
8	0.9571		.0056
		.0447	
9	0.9124		.0056
		.0503	
10	0.8621		.0056
		.0559	
11	0.8062		.0056
		.0615	
12	0.7447		.0056
		.0671	
13	0.6776		.0056
		.0727	
14	0.6049		.0056
		.0783	
15	0.5266		.0056
		.0839	
16	0.4427		.0056
		.0895	
17	0.3532		.0056
		.0951	
18	0.2581		.0056
		.1007	
19	0.1574		.0056
		.1063	
20	0.0511		
	14.8350		

The Numbers in the second Column of this Table shew the Content of every Inch of the half Spheroid HhAtH, the Sum of these is 14.835, this doubled is 29.67 the Content of the whole, which is not a quarter of a Pint less than the Content found by the former Rule, for the Area of the Circle HT is 1.114, this Multiplied by $\frac{2}{3}$ of AE, *viz.* 26.66 gives 29.699.

Note,

If Sb be 14 Inches, we may find the Content of the Fruſtum HhtH, by adding the firſt 14 Numbers in the ſecond Column of the foregoing Table together, the Sum will be 13.0459 Ale-Gallons, and this doubled is 26.0918 the Content of the middle Fruſtum htTPg, which may repreſent a Cask, as the half of it may repreſent a Tun, and if a Tun be in this form, having for one of its Baſes the greateſt Circle of the Spheroid, the Content may be found as in Problem I. Sect. IX. of the foregoing Tract, and by the VII. Problem of the ſame Section, the Diameters, and conſequently the Content upon every Inch of ſuch Tun may be readily found.

In the Definitions of the Solids here treated of, they are ſaid to be made by a plain figure, being turned about a Right Line given; and theſe Solids do all of them (except the Spheroid) take their denomination from the Figure turned about, as Parabolick Conoids, and Spindles from Parabola's, Hyperbolicks from Hyperbola's, and even of the Spheroid it ſelf if a part be taken towards the Vertex A, it is by ſome called an Elliptical Conoid.

Now if the Area or Content of any plain Figure, and the diſtance of its Center of Gravity from the Line about which it is turned be given, the Content of the Solid deſcribed by the turning about of ſuch a plain Figure is alſo given; for,

If a Plain Figure, turned about a Right Line given in the ſame Plain (which ſhall not cut the Figure given) ſhall deſcribe a Solid Figure: this

Solid

Solid is equal to a Prismatick Solid upon the same or an equal Base, having Altitude equal to the Perifery (perfect or imperfect, as the Conversion was perfect or imperfect) which is described by the Center of Gravity of the Figure given to be turned. See this demonstrated by Dr. *Wallis* in his *Mechanica*, page 197. who hath shewn in the same Book how to find the Center of Gravity of any Plain Figure, and proved that the Center of Gravity of a Semi-Parabola (suppose A P o in Fig. IV.) is distant from the Vertex A, $\frac{3}{5}$ of the Altitude, and from the Line A B, $\frac{3}{8}$ of the Latitude, that is, $\frac{3}{8}$ of P o, therefore the Center of Gravity of the Semi-Parabola, will be in the point a; and of the whole Parabola in the point x.

Now if the Parallelogram A N o P, be turned about the Line A P, it will describe the Cylinder N o o V, and by the same Conversion the Semi-Parabola A O P, will describe the Parabolick Conoid A o P o, upon the same Base o o having Altitude A P, the same with the Cylinder, and from the proportion of the Semi-Parabola, to the Parallelogram Circumscribed, and from the Center of Gravity of one to that of the other is proved, that the Parabolick Conoid A o P o, to the Cylinder N o o V is as 1 to 2, that is, the former is half of the latter. Dr. *Wallis's Mechanica*, Part 2. Problem 12.

Now whilst the Figure is turning about the Center of Gravity a, will describe a Circumference, whose Radius is x a, and the length of this Circumference Multiplied by the Area of the Semi Parabola A o P, produces the Content of the Parabolick Conoid, o A o P,

Moreover,

Moreover if the Parallelogram T K H G, (Fig. 12.) be turned about the Line G H, it will describe the Cylinder I K M L, and the Parabola G A H (whose Center of Gravity x, is distant from the Vertex A, $\frac{2}{3}$ of the Altitude A Q) will describe the Parabolick Spindle G A H B, which Parabolick Spindle is to the Cylinder Circumscribed as 8 to 15, as is proved in the place above cited.

Now whilst the Figure is turning about, the Center of Gravity x, will describe a Circumference, whose Radius Q x being given, the length of the Circumference is also given, which Multiplied by the Area of the Parabola G A H, the Product is the Content of the Parabolick Spindle G A H B.

I might now compare these Rules with those I have given above, but I will first give an Example in two Solids more commonly known, viz. a Cylinder and a Sphere, now it is well known to the meanest Geometrician, that a Sphere to a Cylinder of the same Base and Altitude is as 2 to 3. Suppose the Diameter and Altitude of a Cylinder be each 12 Inches, the Content will be found to be 1357.1539 Solid Inches; $\frac{2}{3}$ of this is 904.7692 the Content of a Sphere whose Diameter is 12 Inches.

If B D (Fig. 9.) be 12, and A C 6, the Area of the Parallelogram B E F D is 72; the Center of Gravity is in the middle at v, so the Radius A v being 3 the length of the Circumference v n r e (which is described by the point v, whilst the Parallelogram is turning about) is 18.84936, this Multiplied by 72 (the Area of the Parallelogram) gives 1357.1539 the Content of the Cylinder as above.

Again,

Again, the Center of Gravity of a Semi-Circle (according to Dr. *Wallis*) is thus found,

As the Semi-Periphery is to the Diameter,
So is $\frac{2}{3}$ of the Radius, to the distance of the Center of Gravity from the Center of the Circle.

In this example the Diameter B D is 12, and the Semi-Periphery B C D, 18.8495, therefore

As 18.8495 the Semi-Periphery, to 12 the Diameter,
So is $\frac{2}{3}$ of the Radius, viz. 4 to 2.54649, which is equal to A S, the distance of the Center of Gravity from the Center of the Circle. Now whilst the Semi-Circle is turning about, the Center of Gravity S, will describe the Circumference S z x c, whose length is 16 Inches, because the Radius A S is 2.54649.

The Area of the whole Circle is 183.09616, the half is 56.54808 the Area of the Semi-Circle B C D, this Multiplied by 16, gives 904.76948 the Content of the Sphere, which is but $\frac{11}{100000}$ of a Cubick Inch more than the Content found as above.

If this Rule be applied to the other Solids, the like agreement will appear; for instance,

In Fig. IV. P o is 20 Inches, and P A 40, therefore the Parallelogram N o P A is 800; $\frac{1}{3}$ of this is 333.333 the Area of the Semi-Parabola o o A P.

The Center of Gravity a, is distance from A P $\frac{1}{3}$ of P o, therefore x a is 7.5. Now if the Semi-Parabola be turned about the Line A P, the Center of Gravity a, will describe a Circumference whose length will be 47.1248, this Multiplied by the Area of the Semi-

Semi-Parabola, *viz.* 333.333 gives 25132.784, the Content of the Parabolick Conoid, whose Base is o P o, and Altitude P A, and this is half the Circumscribed Cylinder N o o V, for the Diameter o P o is 40 Inches, therefore the Area of the Base is 1256.6384, this Multiplied by P A the Altitude, *viz.* 40, gives 50265.536 the Content of the Cylinder, the half is 25132.768, the difference is but $\frac{16}{100}$ of a Cubick Inch, and there would be no difference at all if the Work could be performed by whole Numbers. I might further shew the agreement of these Rules, in the Parabolick Spindle, and other Solids, but by what hath been already said, I presume the method is so plain as not to need more Examples.

The END.

Fig. II

Fig. III.

ADVERTISEMENT.

THE Church History clear'd from the *Roman* Forgeries and Corruptions found in the Councils and *Baronius*; In Four Parts. From the beginning of Christianity, to the end of the Fifth General Council, 553. By *Thomas Comber*, D.D. Dean of *Durham*.

A Discourse on the Offices for the Fifth of *November*, Thirtieth of *January*, and Twenty Ninth of *May*. By *Thomas Comber*, D.D. Dean of *Durham*. Both Printed for *Robert Clavel*, at the *Peacock* in S. *Paul's Church-Yard*.

ADVERTISEMENT.

AN Idea of Geography and Navigation. Containing easie Rules for finding the Latitude and difference of Longitude of Places by Observation of the Sun, Moon and Stars. The Demonstration and Use of the Log-Line. The variation of the Compass. The Doctrine of Plain Triangles. The Construction and Use of all manner of Maps and Charts. To keep a Journal, and to work a Traverse both by Plain and *Mercator's* Sailing. The Solution of all Nautical Questions, Geometrically, Arithmetically, and Instrumentally. Also Tables of the Sun's Declination and Right Ascension for ever. A Table of the most Eminent fixed Stars in both Hemispheres, rectified for the Year 1700, with their Use, and other Tables necessary in Navigation. By *Samuel Newton*, Master of the Mathematical School at *Christ's Hospital*, Founded by King *Charles* II. Printed for *Christopher Hussey*, at the *Flower de Luce* in *Little Britain*.

Wm Mitchell
His Book
Decembery 10
1763

Jacobo : Scott
& hugos : Libro
1° : D : 1713